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On Three Dimensional Kenmotsu Manifolds admitting A quater symmetric metric connection

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Abstract. The object of the present paper is to study a type of quater-symmetric metric connection in a 3-dimensional Kenmotsu manifold.We study some curvature properties of a 3-dimensional Kenmotsu manifold with respect to the quater- Symmetric metric connection.Finally,we construct an example of a 3-dimensional Kenmotsu manifold to verify the results.

Key Words and Phrases: Quater-symmetric metric connection, Kenmotsu manifold,η-parallel Ricci tensor, cyclic Ricci tensor,locally ϕ-symmetric manifold

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1. Introduction

The product of an almost contact manifold M and the real line $\mathbb{R}$ carries a natural almost complex structure. However if one takes M to be an almost contact metric manifold and suppose that the product metric G on $M \times \mathbb{R}$ is Kaehlerian, then the structure on M is cosymplectic [9] and not Sasakian. On the other hand, Oubina [12] pointed out that if the conformally related metric e^{2t}G, t being the coordinates on $\mathbb{R}$, is Kaehlerian, then M is Sasakian and conversely.

In [19] S. Tanno classified almost contact metric manifolds whose automorphism groups possesses the maximum dimension. For such a manifold M ,the sectional curvature of a plane section containing ξ is a constant ,say c.If c> 0,M is a homogeneous Sasakian manifold of constant sectional curvature. If c= 0,M is the product of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature. If c< 0, M is a warped product space $\mathbb{R} \times f^{C^n}$. In 1972 , K. Kenmotsu [10] abstracted the differential geometric properties of the third case. We call it Kenmotsu manifold.

The quater-symmetric connection generalizes the semi-symmetric connection. The semi-symmetric metric connection is important in the geometry of Riemannian manifolds having also physical application; for instance, the displacement on the earth surface following a fixed point is metric and semi-symmetric [17].

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In this paper we undertake a study of quater-symmetric metric connection in a 3-dimensional Kenmotsu manifold.In 1975 , S.Golab [7] defined and studied quater-symmetric connection in a differentiable manifold.

A linear connection$\widetilde{\nabla}$on an n-dimensional Riemannian manifold (M, g) is called a quater-symmetric connection [7] if its torsion tensor T of the connection $\widetilde{\nabla}$ defined by $T(X, Y) = \widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X - [X, Y]$, satisfies

$$T(X, Y) = \eta(Y)\phi X - \eta(X)\phi Y, \tag{1.1}$$

where η is a 1-form and ϕ is a $(1, 1)$ tensor field.

In particular, if $\phi(X) = X$, then the quater-symmetric connection reduces to a semi-symmetric connection [6].
If moreover,a quater-symmetric connection $\widetilde{\nabla}$satisfies the condition

$$(\widetilde{\nabla}_X g)(Y, Z) = 0, \tag{1.2}$$

for all X,Y,Z ε T(M), where T(M) is the Lie algebra of vector fields of the manifold M,then $\widetilde{\nabla}$ is said to be a quater-symmetric metric connection, otherwise it is said to be a quater-symmetric non-metric connection.
After S.Golab [7] , S.C. Rastogi ([13],[14]) continued the systematic study of quater-symmetric metric connection. In 1980,R.S.Mishra and S.N.Pandey [11] studied quater-symmetric metric connection in Riemannian,Kaehlerian and Sasakian manifolds.In 1982, K.Yano and T.Imai [20] studied quater-symmetric metric connection in Hermition and Kaehlerian manifolds.In 1991, S. Mukhopadhyay, A.K. Roy and B. Barua [15] studied quater-symmetric metric connection on a Riemannian manifold (M,g) with an almost complex structure ϕ.In 1997, U.C. De and S.C Biswas [1] studied quater-symmetric metric connection on an SP-Sasakian manifold.In 2008,U.C.De and A.K. Mondal studied quater-symmetric metric connection on a Sasakian manifold [4].Also in 2008, Sular,Ozgur and De [16] studied quater-symmetric metric connection in a Kenmotsu manifold. The paper is organized as follows:

In section 2, some preliminary results are recalled.After preliminaries, we study 3-dimensional Kenmotsu manifold with η -parallel Ricci tensor and cyclic parallel Ricci tensor with respect to the quater-symmetric metric connection.In the next section ,we characterize locally ϕ-symmetric Kenmotsu manifold with respect to the quater-symmetric metric connection.Finally we construct an example of a 3-dimensional Kenmotsu manifold .

2. Preliminaries

Let M be a connected almost contact metric manifold with an almost contact metric structure (ϕ,ξ,η,g),that is, ϕ is an (1,1) tensor field, ξ is a vector field ,η is a 1 - form and g is a compatible Riemannian metric such that

$$\phi^2(X) = -X + \eta(X)\xi, \eta(\xi) = 1, \phi\xi = 0, \eta\phi = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.2}$$

$$g(X, \xi) = \eta(X), \tag{2.3}$$

for all X,Y ϵ T(M)([2],[3]).
If an almost contact metric manifold satisfies

$$(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{2.4}$$

then M is called a Kenmotsu manifold [10], where ∇ is the Levi-Civita connection of g.From the above equation it follows that

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.5}$$

and

$$(\nabla_X \eta)Y = g(X, Y) - \eta(X)\eta(Y). \tag{2.6}$$

Moreover, the curvature tensor R and the Ricci tensor S satisfy

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.7}$$

and

$$S(X, \xi) = -(n-1)\eta(X). \tag{2.8}$$

From [5] we know that for a 3-dimensional Kenmotsu manifold

$$\begin{aligned} R(X,Y)Z &= (\frac{r+4}{2})[g(Y,Z)X - g(X,Z)Y] - \\ &-(\frac{r+6}{2})[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi + \\ &+\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y], \end{aligned} \tag{2.9}$$

$$S(X, Y) = \frac{1}{2}[(r+2)g(X, Y) - (r+6)\eta(X)\eta(Y)], \tag{2.10}$$

where S is the Ricci tensor of type (0,2), R is the curvature tensor of type (1,3) and r is the scalar curvature of the manifold M.

3. η- parallel Ricci tensor

Definition 1. *The Ricci tensor S of a Kenmotsu manifold is said to be η-parallel if it satisfies*

$$(\nabla_X S)(\phi Y, \phi Z) = 0, \tag{3.1}$$

for all vector fields X, Y and Z.

Let M be a 3-dimensional Kenmotsu manifold. From [16] we know that for a quater-symmetric metric connection in a Kenmotu manifold

$$\widetilde{\nabla}_X Y = \nabla_X Y - \eta(X)\phi Y, \tag{3.2}$$

and

$$\widetilde{S}(Y, Z) = S(Y, Z) + g(\phi Y, Z), \tag{3.3}$$

where $\widetilde{\nabla}$ be a quater-symmetric metric connection in M and $\widetilde{S}$ is the Ricci tensor of the connection $\widetilde{\nabla}$.
We know that

$$\begin{aligned}(\widetilde{\nabla}_X \widetilde{S})(Y, Z) &= \widetilde{\nabla}_X \widetilde{S}(Y, Z) - \\ &\quad - \widetilde{S}(\widetilde{\nabla}_X Y, Z) - \widetilde{S}(Y, \widetilde{\nabla}_X Z).\end{aligned} \tag{3.4}$$

Using (3.2) and (3.3) from (3.4), we have

$$\begin{aligned}(\widetilde{\nabla}_X \widetilde{S})(Y, Z) &= \nabla_X S(Y, Z) + \nabla_X g(\phi Y, Z) - S(\nabla_X Y, Z) + \\ &\quad + \eta(X)S(\phi Y, Z) - g(\phi \nabla_X Y, Z) + \\ &\quad + \eta(X)g(\phi^2 Y, Z) - S(Y, \nabla_X Z) + \\ &\quad + \eta(X)S(Y, \phi Z) - g(\phi Y, \nabla_X Z) + \\ &\quad + \eta(X)g(\phi Y, \phi Z).\end{aligned} \tag{3.5}$$

Now using (2.1),(3.5)yields

$$\begin{aligned}(\widetilde{\nabla}_X \widetilde{S})(Y, Z) &= (\nabla_X S)(Y, Z) + \eta(X)[S(\phi Y, Z) + S(Y, \phi Z)] + \\ &\quad + \eta(Z)g(\phi X, Y) - \eta(Y)g(\phi X, Z).\end{aligned} \tag{3.6}$$

In (3.6) replacing Y by ϕY,Z by ϕZ and using (2.1)we get

$$\begin{aligned}(\widetilde{\nabla}_X \widetilde{S})(\phi Y, \phi Z) &= (\nabla_X S)(\phi Y, \phi Z) + \\ &\quad + \eta(X)[-S(Y, \phi Z) + \eta(Y)S(\xi, \phi Z) - \\ &\quad - S(\phi Y, Z) + \eta(Z)S(\phi Y, \xi)].\end{aligned} \tag{3.7}$$

Now using (2.10),(3.7) yields

$$(\widetilde{\nabla}_X \widetilde{S})(\phi Y, \phi Z) = (\nabla_X S)(\phi Y, \phi Z). \tag{3.8}$$

Hence we can state the following:

Theorem 1. *In a 3-dimensional Kenmotsu manifold, η-parallelity of the Ricci tensor with respect to the quater-symmetric metric connection and the Levi-Civita connection are equivalent.*

4. Cyclic Parallel Ricci tensor

A.Gray [8] introduced two classes of Riemannian manifolds determined by the covariant differentiation of the Ricci tensor.The class A consisting of all Riemannian manifolds whose Ricci tensor S is a Codazzi tensor, that is

$$(\nabla_X S)(Y,Z) = (\nabla_Y S)(X,Z).$$

The class B consisting of all Riemannian manifolds whose Ricci tensor is cyclic parallel,that is, $(\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y) = 0$.
Let M be a 3-dimensional Kenmotsu manifold.Then its Ricci tensor $\widetilde{S}$ is given by (3.3).Now using (3.6)we have

$$\begin{aligned}(\widetilde{\nabla}_X \widetilde{S})(Y,Z) + (\widetilde{\nabla}_Y \widetilde{S})(Z,X) + (\widetilde{\nabla}_Z \widetilde{S})(X,Y) = \\ = (\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y) + \\ +\eta(X)[S(\phi Y,Z) + S(Y,\phi Z)] + \\ +\eta(Y)[S(\phi Z,X) + S(Z,\phi X)] + \\ +\eta(Z)[S(\phi X,Y) + S(X,\phi Y)].\end{aligned} \tag{4.1}$$

Now using (2.10),(4.1)yields

$$\begin{aligned}(\widetilde{\nabla}_X \widetilde{S})(Y,Z) + (\widetilde{\nabla}_Y \widetilde{S})(Z,X) + (\widetilde{\nabla}_Z \widetilde{S})(X,Y) = \\ = (\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y).\end{aligned} \tag{4.2}$$

Hence we can state the following:

Theorem 2. *Cyclic Ricci tensor of a 3-dimensional Kenmotsu manifold with respect to the quater-symmetric metric connection and the Levi-Civita connection are equivalent.*

5. Locally ϕ-symmetric Kenmotsu manifolds

Definition 2. *A Sasakian manifold is said to be locally ϕ-symmetric if*

$$\phi^2(\nabla_W R)(X,Y)Z = 0, \tag{5.1}$$

for all vector fields W,X,Y,Z orthogonal to ξ.

This notion was introduced for Sasakian manifolds by Takahashi [18].
Analogous to the definition of ϕ-symmetric Sasakian manifold with respect to the Riemannian connection,we define locally ϕ-symmetric Kenmotsu manifold with respect to the quater-symmetric metric connection by

$$\phi^2(\widetilde{\nabla}_W \widetilde{R})(X,Y)Z = 0, \tag{5.2}$$

for all vector fields W,X,Y,Z orthogonal to ξ.
Using (3.2) we can write

$$(\widetilde{\nabla}_W \widetilde{R})(X,Y)Z = (\nabla_W \widetilde{R})(X,Y)Z - \eta(W)\phi\widetilde{R}(X,Y)Z. \tag{5.3}$$

From [17] we know that for a Kenmotsu manifold

$$\widetilde{R}(X,Y)Z = R(X,Y)Z + \eta(X)g(\phi Y,Z)\xi - \\ -\eta(Y)g(\phi X,Z)\xi - \eta(X)\eta(Z)\phi Y + \\ +\eta(Y)\eta(Z)\phi X. \tag{5.4}$$

Using (2.9),(5.4) yields

$$\widetilde{R}(X,Y)Z = (\frac{r+4}{2})[g(Y,Z)X - g(X,Z)Y] - \\ -(\frac{r+6}{2})[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi + \\ +\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] + \\ +\eta(X)g(\phi Y,Z)\xi - \\ -\eta(Y)g(\phi X,Z)\xi - \eta(X)\eta(Z)\phi Y + \\ +\eta(Y)\eta(Z)\phi X. \tag{5.5}$$

Now differentiating (5.5) with respect to W and using (2.4),we get from (5.3)

$$(\widetilde{\nabla}_W\widetilde{R})(X,Y)Z = \frac{dr(W)}{2}[g(Y,Z)X - g(X,Z)Y] - \\ -\frac{dr(W)}{2}[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi + \\ +\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] - \\ -(\frac{r+6}{2})[g(Y,Z)(\nabla_W\eta)(X)\xi - g(X,Z)(\nabla_W\eta)(Y)\xi + \\ +g(Y,Z)\eta(X)\nabla_W\xi - g(X,Z)\eta(Y)\nabla_W\xi + \\ +(\nabla_W\eta)(Y)\eta(Z)X + \eta(Y)(\nabla_W\eta)(Z)X - \\ -(\nabla_W\eta)(X)\eta(Z)Y - \eta(X)(\nabla_W\eta)(Z)Y] + \\ +(\nabla_W\eta)(X)g(\phi Y,Z)\xi + \eta(X)g(\phi Y,Z)W - \\ -\eta(X)g(\phi Y,Z)\eta(W)\xi - (\nabla_W\eta)(Y)g(\phi X,Z)\xi - \\ -\eta(Y)g(\phi X,Z)W + \eta(Y)g(\phi X,Z)\eta(W)\xi - \\ -g(W,X)\eta(Z)\phi Y + 2\eta(W)\eta(X)\eta(Z)\phi Y - \\ -\eta(X)g(W,Z)\phi Y - \eta(X)\eta(Z)g(\phi W,Y)\xi + \\ +g(W,Y)\eta(Z)\phi X - 2\eta(W)\eta(Y)\eta(Z)\phi X + \\ +\eta(Y)g(W,Z)\phi X + \eta(Y)\eta(Z)g(\phi W,X)\xi - \\ -\eta(W)\phi\widetilde{R}(X,Y)Z. \tag{5.6}$$

Now taking W,X,Y,Z are horizontal vector fields, that is, W,X,Y,Z are orthogonal to ξ,then we get from the above

$$\phi^2(\widetilde{\nabla}_W\widetilde{R})(X,Y)Z = -\frac{dr(W)}{2}[g(Y,Z)X - g(X,Z)Y]. \tag{5.7}$$

Hence we can state the following :

Theorem 3. *A 3-dimensional Kenmotsu manifold is locally ϕ-symmetric with respect to the quater-symmetric connection if and only if the scalar curvature r is constant.*

6. Example of a 3-dimensional Kenmotsu manifold

We consider the 3-dimensional manifold $M = \{(x, y, z)\varepsilon\mathbb{R}^3, z \neq 0\}$, where (x, y, z) are standard coordinate of $\mathbb{R}^3$.

The vector fields

$$e_1 = z\frac{\partial}{\partial x}, \quad e_2 = z\frac{\partial}{\partial y}, \quad e_3 = -z\frac{\partial}{\partial z},$$

are linearly independent at each point of M.

Let g be the Riemannian metric defined by

$$g(e_1, e_3) = g(e_1, e_2) = g(e_2, e_3) = 0,$$

$$g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = 1.$$

Let η be the 1-form defined by $\eta(Z) = g(Z, e_3)$ for any $Z\varepsilon\chi(M)$. Let ϕ be the $(1, 1)$ tensor field defined by

$$\phi(e_1) = -e_2, \quad \phi(e_2) = e_1, \quad \phi(e_3) = 0.$$

Then using the linearity of ϕ and g, we have

$$\eta(e_3) = 1,$$

$$\phi^2 Z = -Z + \eta(Z)e_3,$$

$$g(\phi Z, \phi W) = g(Z, W) - \eta(Z)\eta(W),$$

for any $Z, W\varepsilon\chi(M)$. Then for $e_3 = \xi$, the structure (ϕ, ξ, η, g) defines an almost contact metric structure on M.

Let ∇ be the Levi-Civita connection with respect to metric g. Then we have

$$\begin{aligned}
[e_1, e_3] &= e_1 e_3 - e_3 e_1 = \\
&= z\frac{\partial}{\partial x}(-z\frac{\partial}{\partial z}) - (-z\frac{\partial}{\partial z})(z\frac{\partial}{\partial x}) = \\
&= -z^2\frac{\partial^2}{\partial x \partial z} + z^2\frac{\partial^2}{\partial z \partial x} + z\frac{\partial}{\partial x} = \\
&= e_1.
\end{aligned}$$

Similarly

$$[e_1, e_2] = 0 \quad \textit{and} \quad [e_2, e_3] = e_2.$$

The Riemannian connection ∇ of the metric g is given by

$$2g(\nabla_X Y, Z) = Xg(Y, Z) + Yg(Z, X) - Zg(X, Y) -$$

$$-g(X,[Y,Z]) - g(Y,[X,Z]) + g(Z,[X,Y]), \tag{6.1}$$

which known as Koszul's formula.

Using (6.1) we have

$$\begin{aligned} 2g(\nabla_{e_1}e_3, e_1) = -2g(e_1, -e_1) = \\ = 2g(e_1, e_1). \end{aligned} \tag{6.2}$$

Again by (6.1)

$$2g(\nabla_{e_1}e_3, e_2) = 0 = 2g(e_1, e_2), \tag{6.3}$$

and

$$2g(\nabla_{e_1}e_3, e_3) = 0 = 2g(e_1, e_3). \tag{6.4}$$

From (6.2), (6.3) and (6.4) we obtain

$$2g(\nabla_{e_1}e_3, X) = 2g(e_1, X),$$

for all $X \varepsilon \chi(M)$. Thus

$$\nabla_{e_1}e_3 = e_1.$$

Therefore, (6.1) further yields

$$\begin{gathered} \nabla_{e_1}e_3 = e_1, \quad \nabla_{e_1}e_2 = 0, \quad \nabla_{e_1}e_1 = -e_3, \\ \nabla_{e_2}e_3 = e_2, \quad \nabla_{e_2}e_2 = e_3, \quad \nabla_{e_2}e_1 = 0, \\ \nabla_{e_3}e_3 = 0, \quad \nabla_{e_3}e_2 = 0, \quad \nabla_{e_3}e_1 = 0. \end{gathered} \tag{6.5}$$

From the above it follows that the manifold satisfies $\nabla_X \xi = X - \eta(X)\xi$, for $\xi = e_3$.Hence the manifold is a Kenmotsu manifold. It is known that

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]}Z. \tag{6.6}$$

With the help of the above results and using (6.6), it can be easily verified that

$$\begin{gathered} R(e_1,e_2)e_3 = 0, \quad R(e_2,e_3)e_3 = -e_2, \quad R(e_1,e_3)e_3 = -e_1, \\ R(e_1,e_2)e_2 = -e_1, \quad R(e_2,e_3)e_2 = e_3, \quad R(e_1,e_3)e_2 = 0, \\ R(e_1,e_2)e_1 = e_2, \quad R(e_2,e_3)e_1 = 0, \quad R(e_1,e_3)e_1 = e_3. \end{gathered}$$

From the equation (3.2),we find

$$\begin{gathered} \widetilde{\nabla}_{e_1}e_3 = e_1, \quad \widetilde{\nabla}_{e_1}e_2 = 0, \quad \widetilde{\nabla}_{e_1}e_1 = -e_3, \\ \widetilde{\nabla}_{e_2}e_3 = e_2, \quad \widetilde{\nabla}_{e_2}e_2 = -e_3, \quad \widetilde{\nabla}_{e_2}e_1 = 0, \\ \widetilde{\nabla}_{e_3}e_3 = 0, \quad \widetilde{\nabla}_{e_3}e_2 = -e_1, \quad \widetilde{\nabla}_{e_3}e_1 = e_2. \end{gathered} \tag{6.7}$$

Using (5.4)in (6.7),we obtain

$$\widetilde{R}(e_1,e_2)e_3 = 0, \quad \widetilde{R}(e_2,e_3)e_3 = -e_2, \quad \widetilde{R}(e_1,e_3)e_3 = -e_1,$$

$$\widetilde{R}(e_1,e_2)e_2 = -e_1, \quad \widetilde{R}(e_2,e_3)e_2 = e_3, \quad \widetilde{R}(e_1,e_3)e_2 = e_3,$$

$$\widetilde{R}(e_1,e_2)e_1 = e_2, \quad \widetilde{R}(e_2,e_3)e_1 = -e_3, \quad \widetilde{R}(e_1,e_3)e_1 = e_3.$$

From the above expressions of the curvature tensor R we obtain

$$S(e_1,e_1) = g(R(e_1,e_2)e_2,e_1) + g(R(e_1,e_3)e_3,e_1) = \\ = -2.$$

Similarly, we have

$$S(e_2,e_2) = S(e_3,e_3) = -2.$$

Therefore

$$r = S(e_1,e_1) + S(e_2,e_2) + S(e_3,e_3) = -6.$$

Again from the above expressions of the curvature tensor $\widetilde{R}$ we obtain

$$\widetilde{S}(e_1,e_1) = g(\widetilde{R}(e_1,e_2)e_2,e_1) + g(\widetilde{R}(e_1,e_3)e_3,e_1) = \\ = -2.$$

Similarly, we have

$$\widetilde{S}(e_2,e_2) = \widetilde{S}(e_3,e_3) = -2.$$

Therefore

$$\widetilde{r} = \widetilde{S}(e_1,e_1) + \widetilde{S}(e_2,e_2) + \widetilde{S}(e_3,e_3) = -6.$$

We note that here r and $\widetilde{r}$ are all constants.Since S=$\widetilde{S} = -2$,therefore theorem 1 and 2 are verified.

From the expressions of $\widetilde{R}$,it follows that $\phi^2(\widetilde{\nabla}_W \widetilde{R})(X,Y)Z = 0$.Also the scalar curvature is constant.Hence the 3- dimensional Kenmotsu manifold is locally ϕ-symmetric.Thus Theorem 3 is verified.

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