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A Review on Unbiased Estimators for a Parameter of Morgenstern Type Bivariate Gamma Distribution Using Ranked Set Sampling

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Abstract. We obtain unbiased estimators for a parameter of Morgenstern type bivariate gamma distribution (MTBGD) based on the observations made on the units of the ranked set sampling regarding the study variable Y which is correlated with the auxiliary variable X, when (X, Y) follows a MTBGD. Efficiency comparisons among these estimators are also made in this work. Finally, we illustrate the methods developed by using a real data set in marine biological science.

Key Words and Phrases: Concomitants of order statistics, Morgenstern family, Ranked set sampling, Relative efficiency

2010 Mathematics Subject Classifications: 62D05; 62F07

1. Introduction

Ranked set sampling (RSS) was first proposed by [9] for estimating the mean pasture yields. McIntyre indicates that RSS is a more efficient sampling method than simple random sampling (SRS) method for estimating the population mean. In the RSS technique, the sample selection procedure is composed of two stages. At the first stage of sample selection, n simple random samples of size n are drawn from an infinite population and each sample is called a set. Then, each of units are ranked from the smallest to the largest according to variable of interest, say X, in each set. Ranking of the units is done with a low-level measurement such as using previous experiences, visual measurement or using a concomitant variable. At the second stage, the first unit from the first set, the second unit from the second set and going on like this nth unit from the nth set are taken and measured according to the variable X with a high level of measurement satisfying the desired sensitivity. The obtained sample is called an RSS.

[14] introduced a modified ranked set sampling procedure in which only the largest or the smallest ranked unit is chosen for quantification. [11] investigated the use of a variety of extreme ranked set samples (ERSS) for estimating the population mean. Another scheme of ranked set sampling was investigated by [1] which is the moving extreme ranked set

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sampling (MERSS). It is a modification of the RSS where only the lowest or largest unit of sets of varied sizes is measured. [13] applied RSS for bivariate random variable (X, Y), where X is the variable of interest and Y is a concomitant variable that is not of direct interest but is relatively easy to measure. Let $X_{(r)r}$ be the measured observation on the variable X in the rth unit of the RSS and let $Y_{[r]r}$ be the corresponding measurement made on the study variable Y of the same unit, $r = 1, 2, 3, \cdots, n$. Then clearly $Y_{[r]r}$ is the concomitant of rth order statistic arising from the rth sample.

[8] used RSS to estimate the two-parameter exponential distribution. [15] has obtained the estimation of parameters of location-scale family of distribution by RSS. [2, 3] estimated the means of the bivariate normal distribution using moving extreme RSS with concomitant variable. Estimation of a parameter of Morgenstern type bivariate exponential distribution by using RSS was considered by [5]. [4] considered estimation of the parameters of Downton's bivariate exponential distribution using RSS scheme.

A Morgenstern type bivariate gamma distribution (MTBGD) is constructed by [6] with the probability density function (pdf)

$$f_{X,Y}(x, y) = \frac{x^{\alpha-1} y^{\beta-1} \exp(-x-y)}{\Gamma(\alpha)\Gamma(\beta)} [1 + \lambda(1 - 2P(\alpha, x))(1 - 2P(\beta, y))], \quad x, y > 0, \beta > 0, \ |\lambda| > 0, \tag{1}$$

where $P(a, z) = \frac{1}{\Gamma(a)} \int_0^z e^{-t} t^{a-1} \mathrm{d}t$ is the incomplete gamma function. [7] studied the moments and the correlation coefficient, and [10] discussed the general distribution theory of Morgenstern type bivariate gamma distribution and the properties of the concomitants of order statistics from it and presented estimations for the parameters of the distribution using the concomitants and method of moments.

In [12] the pdf of $Y_{[r]r}$ for $1 \leq r \leq n$ is given by

$$h_{[r]r}(y) = \frac{y^{\beta-1} \exp(-y)}{\Gamma(\beta)} [1 + \lambda(\frac{n - 2r + 1}{n + 1})(1 - 2P(\beta, y))], \quad y > 0, \tag{2}$$

and its mean and variance of $Y_{[r]r}$ are obtained by [10] as

$$E[Y_{[r]r}] = \beta - \delta_r, \quad Var[Y_{[r]r}] = \beta - \delta_r - \delta_r^2, \tag{3}$$

where $\delta_r = \frac{\lambda(n-2r+1)}{4^\beta (n+1) B(\beta, \beta+1)}$ and $B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$.

The organization of this article is as follows: In Section 2, we present four unbiased estimators for a parameter, β in MTBGD when λ and α are known. These estimators are also made by the ranked set sample mean, and the ERSS and MERSS methods. We evaluate the efficiency of all estimators considered in this paper. In Section 3, we illustrate the ERSS and MERSS methods using a real data set in marine biological science.

2. Main Results

Suppose that the random variable (X, Y) has a MTBGD as defined in (1). In this section, we find four unbiased estimators for the parameter β based on different sampling schemes. In each case, we first present the general pattern and then, an unbiased estimators with its variances is given.

2.1. RSS estimation

The procedure of RSS described by [13] for a bivariate random variable is as follows:
Step 1. Randomly select n independent bivariate samples, each of size n.
Step 2. Rank the units within each sample with respect to a variable of interest X together with the Y variate associated.
Step 3. In the rth sample of size n, select the unit $(X_{(r)r}, Y_{[r]r})$, $r = 1, 2, ..., n$.
Therefore, $Y_{[r]r}$, $r = 1, 2, 3, \cdots, n$, are the RSS observations made on the units of the ranked set sampling regarding the study variable Y which is correlated with the auxiliary variable X.

Theorem 1. *An unbiased estimator for β based on the procedure of RSS is given by*

$$\hat{\beta}_{RSS} = \frac{1}{n}\sum_{r=1}^{n} Y_{[r]r}, \tag{4}$$

and its variance is

$$Var(\hat{\beta}_{RSS}) = \frac{\beta}{n}\{1 - \frac{\delta_n^2(n+1)}{3\beta(n-1)}\}, \tag{5}$$

where $\delta_n = \frac{\lambda(1-n)}{4^{\beta}(n+1)B(\beta,\beta+1)}$.

Proof. Since $\sum_{r=1}^{n}\delta_r = 0$, using (3) we have

$$E(\hat{\beta}_{\text{RSS}}) = \frac{1}{n}\sum_{r=1}^{n} E(Y_{[r]r}) = \frac{1}{n}\sum_{r=1}^{n}(\beta - \delta_r) = \beta.$$

Also, since $\sum_{r=1}^{n}(n-2r+1)^2 = \frac{1}{3}n(n+1)(n-1)$, we have

$$\begin{aligned} Var(\hat{\beta}_{\text{RSS}}) &= \frac{1}{n^2}\sum_{r=1}^{n} Var(Y_{[r]r}) = \frac{1}{n^2}\sum_{r=1}^{n}(\beta - \delta_r - \delta_r^2) \\ &= \frac{\beta}{n} - \frac{\lambda^2(n-1)}{3\times 4^{2\beta}(n+1)n[B(\beta,\beta+1)]^2} \\ &= \frac{\beta}{n}\{1 - \frac{(n+1)\delta_n^2}{3\beta(n-1)}\}, \end{aligned}$$

and proof is completed.◀

By comparing the variance of $\hat{\beta}_{RSS}$ with the Cramer Rao lower bound $\frac{\beta}{n}$ of any unbiased estimator of β based on a simple random sample (SRS) of size n from $Gamma(\beta, 1)$, we obtain the efficiency of $\hat{\beta}_{RSS}$ as

$$e_1 = \frac{\frac{\beta}{n}}{\frac{\beta}{n}\{1 - \frac{(n+1)\delta_n^2}{3\beta(n-1)}\}} = \frac{1}{1 - \frac{\lambda^2(n-1)[\Gamma(2\beta)]^2}{3\times 4^{2\beta-1}\beta(n+1)[\Gamma(\beta)]^4}}.$$

Obviously, we can see that i) $e_1 \geq 1$, ii) for fixed $n > 1$, the efficiency increases as $|\lambda|$ increases.

2.2. ERSS Estimation

The extreme ranked set sampling (ERSS) method with concomitant variable introduced by [11] can be described as follows:

Step 1. Select n random samples each of size n, bivariate units from the population.

Step 2. If the sample size n is even, then select from $\frac{n}{2}$ samples the smallest ranked unit X together with the associated Y and from the other $\frac{n}{2}$ samples the largest ranked unit X together with the associated Y. This selected observations $(X_{(1)1}, Y_{[1]1}), (X_{(n)2}, Y_{[n]2}), (X_{(1)3}, Y_{[1]3}), ..., (X_{(1)n-1}, Y_{[1]n-1}), (X_{(n)n}, Y_{[n]n})$ can be denoted by ERSS$_1$.

Step 3. If n is odd, then select from $\frac{n-1}{2}$ samples the smallest ranked unit X together with the associated Y and from the other $\frac{n-1}{2}$ samples the largest ranked unit X together with the associated Y and from one sample the median of the sample for actual measurement. In this case the selected observations $(X_{(1)1}, Y_{[1]1}), (X_{(n)2}, Y_{[n]2}), (X_{(1)3}, Y_{[1]3}), \ldots, (X_{(n)n-1}, Y_{[n]n-1}), (\frac{X_{(1)n}+X_{(n)n}}{2}, \frac{Y_{[1]n}+Y_{[n]n}}{2})$ can be denoted by ERSS$_2$ and $(X_{(1)1}, Y_{[1]1}), (X_{(n)2}, Y_{[n]2}), (X_{(1)3}, Y_{[1]3}), \ldots, (X_{(n)n-1}, Y_{[n]n-1}), (X_{(\frac{n+1}{2})n}, Y_{[\frac{n+1}{2}]n})$ can be denoted by ERSS$_3$.

Theorem 2. *i. If n is even, then an unbiased estimator of β using ERSS$_1$ is*

$$\hat{\beta}_{ERSS_1} = \frac{1}{n}\sum_{r=1}^{n/2}(Y_{[1]2r-1} + Y_{[n]2r}), \tag{6}$$

with variance

$$Var(\hat{\beta}_{ERSS_1}) = \frac{\beta}{n}\{1 - \frac{\delta_n^2}{\beta}\}. \tag{7}$$

ii. If n is odd, then unbiased estimators of β using ERSS$_2$ and ERSS$_3$ are

$$\hat{\beta}_{ERSS_2} = \frac{1}{n}\sum_{r=1}^{(n-1)/2}(Y_{[1]2r-1} + Y_{[n]2r}) + \frac{Y_{[1]n} + Y_{[n]n}}{2n}, \tag{8}$$

$$\hat{\beta}_{ERSS_3} = \frac{1}{n}\sum_{r=1}^{(n-1)/2}(Y_{[1]2r-1} + Y_{[n]2r}) + \frac{Y_{[\frac{n+1}{2}]n}}{n}, \tag{9}$$

with variance

$$Var(\hat{\beta}_{ERSS_2}) = \frac{\beta}{n}\{1 - \frac{\delta_n^2[(2n-1)(n+2)(n-1)^2 - 4]}{2n\beta(n+2)(n-1)^2}\}, \tag{10}$$

$$Var(\hat{\beta}_{ERSS_3}) = \frac{\beta}{n}\{1 - \frac{(n-1)\delta_n^2}{n\beta}\}. \tag{11}$$

respectively.

Proof. i. Since $\sum_{r=1}^{n/2}\delta_1=\frac{\lambda n(n-1)}{2\times 4^{\beta}(n+1)B(\beta,\beta+1)}$ and $\sum_{r=1}^{n/2}\delta_n=\frac{\lambda n(-n+1)}{2\times 4^{\beta}(n+1)B(\beta,\beta+1)}$, we have

$$E(\hat{\beta}_{\mathrm{ERSS}_1})=\frac{1}{n}\sum_{r=1}^{n/2}(E(Y_{[1]2r-1})+E(Y_{[n]2r}))=\frac{1}{n}\sum_{r=1}^{n/2}(\beta-\delta_1+\beta-\delta_n)=\beta.$$

Also, since $\sum_{r=1}^{n/2}\delta_1^2=\sum_{r=1}^{n/2}\delta_n^2=\frac{n\delta_n^2}{2}$, we have

$$\begin{aligned}Var(\hat{\beta}_{\mathrm{ERSS}_1}) &= \frac{1}{n^2}\sum_{r=1}^{n/2}(Var(Y_{[1]2r-1})+Var(Y_{[n]2r}))\\ &= \frac{1}{n^2}\sum_{r=1}^{n/2}(\beta-\delta_1-\delta_1^2+\beta-\delta_n-\delta_n^2)=\frac{\beta}{n}-\frac{\delta_n^2}{n}.\end{aligned}$$

ii. In the estimator $\hat{\beta}_{\mathrm{ERSS}_2}$, it is easy to see that $Y_{[1]1},Y_{[n]2},Y_{[1]3},...,Y_{[n]n-1}$ are independent of $Y_{[1]n}$ and $Y_{[n]n}$, but the random variables $Y_{[1]n}$ and $Y_{[n]n}$ are dependent. From [12] the joint density function of $(Y_{[1]n},Y_{[n]n})$ is given by

$$\begin{aligned}h_{[1,n]n}(z,w) &= \frac{(zw)^{\beta-1}e^{-(z+w)}}{[\Gamma(\beta)]^2}[1+2\lambda(\frac{n-1}{n+1})(P(\beta,w)-P(\beta,z))\\ &\quad -\frac{\lambda^2(n-2)}{n+2}(1-2P(\beta,w))(1-2P(\beta,z))].\end{aligned}$$

Therefore, we have

$$Cov(Y_{[1]n},Y_{[n]n})=\frac{\lambda^2}{(n+1)^2(n+2)4^{2\beta-1}[B(\beta,\beta+1)]^2}.$$

Also, $Y_{[1]1},Y_{[n]2},Y_{[1]3},...,Y_{[n]n-1}$ and $Y_{[\frac{n+1}{2}]n}$ are all independent in $\hat{\mu}_{\mathrm{ERSS}_3}$. Since

$$\sum_{r=1}^{(n-1)/2}\delta_1=\frac{\lambda(n-1)^2}{2\times 4^{\beta}(n+1)B(\beta,\beta+1)},\qquad \sum_{r=1}^{(n-1)/2}\delta_n=\frac{-\lambda(n-1)^2}{2\times 4^{\beta}(n+1)B(\beta,\beta+1)},$$

and $\delta_{(n+1)/2}=0$, we have

$$\begin{aligned}E(\hat{\beta}_{\mathrm{ERSS}_2}) &= \frac{1}{n}\sum_{r=1}^{(n-1)/2}(E(Y_{[1]2r-1})+E(Y_{[n]2r}))+\frac{E(Y_{[1]n})+E(Y_{[n]n})}{2n}\\ &= \frac{1}{n}\sum_{r=1}^{(n-1)/2}(\beta-\delta_1+\beta-\delta_n)+\frac{\beta-\delta_1+\beta-\delta_n}{2n}=\beta,\\ E(\hat{\beta}_{\mathrm{ERSS}_3}) &= \frac{1}{n}\sum_{r=1}^{(n-1)/2}(E(Y_{[1]2r-1})+E(Y_{[n]2r}))+\frac{E(Y_{[\frac{n+1}{2}]n})}{n}\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{n}\sum_{r=1}^{(n-1)/2}(\beta-\delta_1+\beta-\delta_n)+\frac{\beta-\delta_{(n+1)/2}}{n}=\beta,\\
Var(\hat{\beta}_{\mathrm{ERSS}_2}) &= \frac{1}{n^2}\sum_{r=1}^{(n-1)/2}[Var(Y_{[1]2r-1})+Var(Y_{[n]2r})]\\
&+\frac{Var(Y_{[1]n})+Var(Y_{[n]n})+2Cov(Y_{[1]n},Y_{[n]n})}{4n^2}\\
&= \frac{\beta}{n}\{1-\frac{\delta_n^2[(2n-1)(n+2)(n-1)^2-4]}{2n\beta(n+2)(n-1)^2}\},\\
Var(\hat{\beta}_{\mathrm{ERSS}_3}) &= \frac{1}{n^2}\sum_{r=1}^{(n-1)/2}[Var(Y_{[1]2r-1})+Var(Y_{[n]2r})]+\frac{Var(Y_{[\frac{n+1}{2}]n})}{n^2}\\
&= \frac{\beta}{n}\{1-\frac{(n-1)\delta_n^2}{n\beta}\}.
\end{aligned}
$$

◀

Now, by using (5), (7), (10) and (11), the efficiency of $\hat{\beta}_{\mathrm{RSS}}$ relative to the estimators $\hat{\beta}_{\mathrm{ERSS}_1}$, $\hat{\beta}_{\mathrm{ERSS}_2}$ and $\hat{\beta}_{\mathrm{ERSS}_3}$, respectively, are given by

$$
\begin{aligned}
e_2 = e(\hat{\beta}_{\mathrm{ERSS}_1} \mid \hat{\beta}_{\mathrm{RSS}}) &= \frac{1-\frac{\delta_n^2(n+1)}{3\beta(n-1)}}{1-\frac{\delta_n^2}{\beta}},\\
e_3 = e(\hat{\beta}_{\mathrm{ERSS}_2} \mid \hat{\beta}_{\mathrm{RSS}}) &= \frac{1-\frac{\delta_n^2(n+1)}{3\beta(n-1)}}{1-\frac{\delta_n^2[(2n-1)(n+2)(n-1)^2-4]}{2n\beta(n+2)(n-1)^2}},\\
e_4 = e(\hat{\beta}_{\mathrm{ERSS}_3} \mid \hat{\beta}_{\mathrm{RSS}}) &= \frac{1-\frac{\delta_n^2(n+1)}{3\beta(n-1)}}{1-\frac{(n-1)\delta_n^2}{n\beta}}.
\end{aligned}
$$

Note that i) $e_2 \geq 1$, for all n, ii) $e_3 \geq 1$ if $n \geq 3$ and $e_3 \leq 1$ if $n = 2$ iii) $e_4 \geq 1$ if $n \geq 3$ and $e_4 \leq 1$ if $n = 2$. iv) for fixed $n > 1$, e_j's increase in $|\lambda|$ and increase in n for fixed $|\lambda| > 0$. So we conclude that $\hat{\beta}_{\mathrm{ERSS}_1}$, $\hat{\beta}_{\mathrm{ERSS}_2}$ and $\hat{\beta}_{\mathrm{ERSS}_3}$ are more efficient than $\hat{\beta}_{RSS}$.

2.3. MERSS Estimation

[3] proposed the concept of MERSS with concomitant variable for the estimation of the means of the bivariate normal distribution. The procedure of MERSS with concomitant variable in MTBGD is as follows:

Step 1. Select n units each of size n from MTBGD using SRS. Identify by judgment the minimum of each set with respect to the variable X.

Step 2. Repeat step 1, but for the maximum.

Note that the $2n$ pairs of set $\{(X_{(1)r}, Y_{[1]r}), (X_{(n)r}, Y_{[n]r}); r = 1, 2, ..., n\}$ that are obtained using the above procedure, are independent but not identically distributed.

Theorem 3. *An unbiased estimator for β based on MERSS is given by*

$$\hat{\beta}_{MERSS} = \frac{1}{2n}\sum_{r=1}^{n}(Y_{[1]r} + Y_{[n]r}), \tag{12}$$

and its variance is

$$Var(\hat{\beta}_{MERSS}) = \frac{\beta}{n}\{1 - \frac{\delta_n^2}{2\beta}\}. \tag{13}$$

Proof. The proof is similar to proof of Theorem 2, part i.◄

Table 1. Comparing efficiency of estimations for $\beta = 10$.

n	$\lvert\lambda\rvert$	e_1	e_2	e_3	e_4	e_5
3	0.20	1.0021	1.0010	1.0004	1.0000	0.9995
3	0.40	1.0083	1.0042	1.0017	1.0000	0.9979
3	0.60	1.0190	1.0096	1.0038	1.0000	0.9953
3	0.80	1.0342	1.0174	1.0069	1.0000	0.9915
3	1.00	1.0546	1.0280	1.0110	1.0000	0.9865
4	0.20	1.0025	1.0020	1.0014	1.0009	0.9998
4	0.40	1.0100	1.0081	1.0056	1.0035	0.9990
4	0.60	1.0229	1.0186	1.0129	1.0081	0.9977
4	0.80	1.0414	1.0342	1.0237	1.0147	0.9959
4	1.00	1.0662	1.0559	1.0384	1.0237	0.9934
5	0.20	1.0028	1.0028	1.0022	1.0017	1.0000
5	0.40	1.0112	1.0113	1.0089	1.0067	1.0000
5	0.60	1.0255	1.0261	1.0206	1.0155	1.0000
5	0.80	1.0462	1.0484	1.0380	1.0285	1.0000
5	1.00	1.0741	1.0800	1.0624	1.0465	1.0000
6	0.20	1.0030	1.0034	1.0029	1.0023	1.0002
6	0.40	1.0120	1.0139	1.0116	1.0095	1.0009
6	0.60	1.0273	1.0323	1.0270	1.0220	1.0020
6	0.80	1.0497	1.0602	1.0501	1.0406	1.0036
6	1.00	1.0798	1.1004	1.0831	1.0669	1.0057
10	0.20	1.0034	1.0050	1.0045	1.0041	1.0008
10	0.40	1.0137	1.0204	1.0186	1.0169	1.0031
10	0.60	1.0314	1.0479	1.0437	1.0395	1.0072
10	0.80	1.0573	1.0909	1.0826	1.0744	1.0132

10	1.00	1.0925	1.1555	1.1404	1.1259	1.0215
15	0.20	1.0036	1.0059	1.0056	1.0053	1.0011
15	0.40	1.0147	1.0245	1.0231	1.0218	1.0046
15	0.60	1.0337	1.0579	1.0546	1.0514	1.0106
15	0.80	1.0615	1.1111	1.1045	1.0979	1.0196
15	1.00	1.0996	1.1930	1.1807	1.1687	1.0321
20	0.20	1.0038	1.0065	1.0062	1.0060	1.0013
20	0.40	1.0152	1.0268	1.0257	1.0246	1.0055
20	0.60	1.0349	1.0636	1.0609	1.0583	1.0126
20	0.80	1.0637	1.1227	1.1172	1.1119	1.0233
20	1.00	1.1033	1.2152	1.2049	1.1948	1.0383

The efficiency of $\hat{\beta}_{\text{RSS}}$ relative to $\hat{\beta}_{\text{MERSS}}$ is given by

$$e_5 = e(\hat{\beta}_{\text{MERSS}} \mid \hat{\beta}_{\text{RSS}}) = \frac{1 - \frac{\delta_n^2(n+1)}{3\beta(n-1)}}{1 - \frac{\delta_n^2}{2\beta}}.$$

Note that $e_5 > 1$ if $n > 5$; $e_5 = 1$ if $n = 5$; $e_5 < 1$ if $n < 5$.

The efficiency of estimations, e_i's, for different values of n and $|\lambda|$ is given in Table 1. We consider $\beta = 10$. Generally, we can find that $e_2 > e_3 > e_4 > e_5$. Therefore, ERSS_1 is better than other methods. Also, e_i's are slowly increasing functions of β.

3. An Application in Marine Biological Science

In this section, we consider a bivariate data set from a marine biological research in the Persian Gulf relating to 300 hawksbill turtle (eretmochelys imbricata) eggs. The first component X from a bivariate observation represents the weight in gram of the eggs and the second component Y represents diameter in millimeter of the hawksbill turtle eggs. Clearly X can be measured easily but it is somewhat difficult to measure Y. Under the assumption that (X, Y) follows MTBGD, we select 9 random samples each of size 9 from the 300 hawksbill turtle eggs data and rank the sampling units of each sample according to the X variate values (weight of the eggs). Then, we measure the ranked set sample observations $Y_{[r]r}$ corresponding to $X_{(r)r}$. The obtained RSS, ERSS_2, ERSS_3 and MERSS observations are reported in Table 2. The computed values of $\hat{\beta}_{\text{RSS}}$, $\hat{\beta}_{\text{ERSS}_2}$, $\hat{\beta}_{\text{ERSS}_3}$ and $\hat{\beta}_{\text{MERSS}}$ are 37.888, 37.416, 37.443 and 37.222, respectively. We can find that the estimated values for β based on different samplings are close.

Table 2. Obtained RSS, ERSS_2, ERSS_3 and MERSS observations.

	r	1	2	3	4	5	6	7	8	9
RSS	$X_{(r)r}$	18.0	30.5	26.0	38.7	29.6	38.8	36.5	29.4	33.0
	$Y_{[r]r}$	34.0	39.0	35.0	41.0	35.0	40.5	40.0	37.0	39.5
$ERSS_2$	$X_{(1)2r-1}$	18.0	25.3	25.2	31.3					
	$Y_{[1]2r-1}$	34.0	34.0	35.0.	39.0					
	$X_{(n)2r}$	32.6	41.5	40.2	30.1					
	$Y_{[n]2r}$	39.0	42.0	40.5	35.0					
	$X_{(1)n}$									28.5
	$Y_{[1]n}$									37.0
	$X_{(n)n}$									33.0
	$Y_{[n]n}$									39.5
$ERSS_3$	$X_{(1)2r-1}$	18.0	25.3	25.2	31.3					
	$Y_{[1]2r-1}$	34.0	34.0	35.0	39.0					
	$X_{(n)2r}$	32.6	41.5	40.2	30.1					
	$Y_{[n]2r}$	39.0	42.0	40.5	35.0					
	$X_{(\frac{n+1}{2})n}$					32.1				
	$Y_{[\frac{n+1}{2}]n}$					38.5				
MERSS	$X_{(1)r}$	18.0	26.1	25.3	28.2	25.2	25.7	31.3	26.6	28.5
	$Y_{[1]r}$	34.0	35.0	34.0	36.0	35.0	35.0	39.0	36.0	37.0
	$X_{(n)r}$	25.0	32.6	40.0	41.5	31.4	40.2	41.1	30.1	33.0
	$Y_{[n]r}$	37.0	39.0	40.0	42.0	38.0	40.5	38.0	35.0	39.5

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