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Intuitionistic Fuzzy Soft Semi-ideals

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Abstract. In this paper, intuitionistic fuzzy soft sets are applied to semirings. The concept of the intuitionistic fuzzy soft semi-ideal over a semiring is introduced. Intersection, Union, And, Or operations of the intuitionistic fuzzy soft semi-ideals are examined and some basic properties are analysed.

Key Words and Phrases: fuzzy soft set, intuitionistic fuzzy set, intuitionistic fuzzy soft semiring.
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1. Introduction

As a generalization of rings, semirings have been found useful for solving problems in various areas of applied mathematics and information sciences, since the structure of a semiring provides an algebraic framework for modeling and studying these applied areas. Generally, semiring results play an important role in graph theory, optimization theory, theory of discrete event dynamical systems, matrices, generalized fuzzy computation, formal language theory, coding theory, automaton theory, determinants, analysis of computer programs, and so on.

Uncertain or imprecise data are pervasive in many important applications in areas such as economics, engineering, environment, social science, medical science and business management. Uncertain data could be caused by data randomness, limitations of measuring instrument, delayed data updates, information incompleteness, etc. There have been a great amount of research and applications in the literature concerning some special tools like probability theory, (intuitionistic) fuzzy set theory, rough set theory, vague set theory and interval mathematics. However, all of these have theirs advantages as wells as inherent limitations in dealing with uncertainties. One major problem shared by those theories is their incompatibility with the parameterizations tools. Soft set theory was firstly proposed by the Russian researcher, Molodtsov [1], to overcome these difficulties in 1999. Nowadays, research on extensions of soft set theory is progressing rapidly. Maji et al. proposed the concept of fuzzy soft set and gave an application [2]. Moreover, they defined and investigated the notion of intuitionistic fuzzy soft set which is based on combination of the soft set and intuitionistic fuzzy sets [3]. Thereafter, they introduced and

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examined several fundamental notions on soft sets [4]. Roy et al. presented a method of object recognition from an imprecise multiobserver data [5]. Kong et al. pointed out that an example in [6], which was used to validate the above method, is only a special case in tabular representation of fuzzy soft set and the algorithm is incorrect. Yao et al. proposed the concept of fuzzy soft sets and defined some operations on fuzzy soft sets [7]. Feng et al. combined the algebraic structure of semirings with the concept of soft sets [8]. All of these researches are based on the Zadeh's fuzzy set theory [9] which was generalized to intuitionistic fuzzy sets by K. Atanassov [10], [11], [12].

The central theme of our paper is the concept of intuitionistic fuzzyness which has a wide range of applications in the field of science and engineering such as intuitionistic fuzzy metric/normed spaces, statistical convergence, functional equations in intuitionistic fuzzy normed spaces, intuitionistic fuzzy soft semigroups which are based on the notion of intuitionistic fuzzy soft set. The theory of intuitionistic fuzzy normed spaces, which has been frequently studied in recent years, has been introduced by Saadati and Park [11] and examined by various authors such as Mursaleen and Mohiuddine [13]-[18]. Further, the studies about intuitionistic fuzzy normed spaces has been taken forward in the papers [19]-[21]. Also, reviewing the recent studies, Zhou et al. [22] introduced the notion of intuitionistic fuzzy soft ideals over a semigroup and they characterized algebraic properties of this notion.

The purpose of this paper is to deal with the algebraic structure of a semiring by applying intuitionistic fuzzy soft theory. For this, we introduce the notion of an intuitionistic fuzzy soft semi-ideals and study some of its properties.

2. Preliminaries

In this section, for the sake of completeness, we first give some useful definitions.

Definition 1. *A semiring is a set S equipped with two binary operations "$+$" and "$\cdot$" on S called addition and multiplication such that:*

(i) $(S, +)$ is a semigroup,
(ii) $(S, \cdot)$ is a semigroup,
(iii) $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$ for all $a, b, c \in S$.

Definition 2. *A nonempty subset I of a semiring S is called a left (right) ideal of S if I is closed under addition and $SI \subseteq I$ ($IS \subseteq I$). I is an ideal of S if it is both a left and a right ideal of the semiring S.*

Definition 3 ([1])**.** *Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subset E$. A pair (F, A) is called a soft set over U, where F is a mapping given by*

$$F : A \to P(U).$$

In other words, a soft set over U is a parametrized family of subsets of the universe U.

Definition 4 ([2]). *Let U be an initial universe, E be a set of parameters and $FS(U)$ denote the fuzzy power set of U and $A \subset E$. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by*

$$F : A \to FS(U).$$

A fuzzy soft set is a parametrized family of fuzzy subsets of U.

Definition 5 ([10]). *An intuitionistic fuzzy set A in a nonempty set X can be defined as follows:*

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle |\, x \in X\},$$

where $\mu_A(x) : X \to [0, 1]$ and $v_A(x) : X \to [0, 1]$ with the property

$$0 \leq \mu_A(x) + v_A(x) \leq 1\, (\forall\ x \in X)\,.$$

The values $\mu_A(x)$ and $v_A(x)$ denote the degree of membership and non-membership of x to A, respectively.

In general , $\forall\ \alpha \in A \subseteq E$, $\widetilde{F}(\alpha)$ is an intuitionistic fuzzy set on U, which is called the intuitionistic fuzzy set of parameter α. The intuitionistic value $\left\langle \mu_{\widetilde{F}(\alpha)}(x), v_{\widetilde{F}(\alpha)}(x) \right\rangle$ denotes the degree of $x \in U$, corresponding to the parameter α. $\widetilde{F}(\alpha)$ can be written as

$$\widetilde{F}(\alpha) = \left\{ \left\langle x, \mu_{\widetilde{F}(\alpha)}(x), v_{\widetilde{F}(\alpha)}(x) \right\rangle \mid x \in U \right\}.$$

If $\forall\ x \in U$, $\mu_{\widetilde{F}(\alpha)}(x) + v_{\widetilde{F}(\alpha)}(x) = 1,$ then $\widetilde{F}(\alpha)$ degenerates into a fuzzy set; if $\forall\ x \in U$ and $\forall\ \alpha \in A \subseteq E$, $\mu_{\widetilde{F}(\alpha)}(x) + v_{\widetilde{F}(\alpha)}(x) = 1,$ then intuitionistic fuzzy soft set $(\widetilde{F}, A)$ degenerates into a fuzzy soft set.

Definition 6 ([3]). *Let U be an initial universe, E be a set of parameters, $IFS(U)$ denote the intuitionistic fuzzy power set of U and $A \subset E$. A pair $(\widetilde{F}, A)$ is called an intuitionistic fuzzy soft set over U, where F is a mapping given by*

$$F : A \to IFS(U).$$

An intuitionistic fuzzy soft set is a parameterized family of intuitionistic fuzzy subsets of U; a fuzzy soft set is a special case of an intuitionistic fuzzy soft set, because when all the intuitionistic fuzzy subsets of U degenerate into fuzzy subsets, the corresponding intuitionistic fuzzy soft set degenerates into a fuzzy soft set.

Definition 7 ([3]). *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over U. Then $(\widetilde{F}, A)$ is called an intuitionistic fuzzy soft subset of $(\widetilde{G}, B)$ if*

1. *$A \subset B$;*
2. *$\widetilde{F}(\alpha)$ is a intuitionistic fuzzy subset of $\widetilde{G}(\alpha)$, for all $\alpha \in A$.*

We denote the above inclusion relationship by $(\widetilde{F}, A)\widetilde{\subseteq}(\widetilde{G}, B)$. Similarly, $(\widetilde{F}, A)$ is called an intuitionistic fuzzy soft superset of $(\widetilde{G}, B)$ if $(\widetilde{G}, B)$ is an intuitionistic fuzzy soft subset of $(\widetilde{F}, A)$ and we denote the above relationship by $(\widetilde{F}, A)\widetilde{\supseteq}(\widetilde{G}, B)$. $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ over a universe U are intuitionistic fuzzy soft equal if $(\widetilde{F}, A)\widetilde{\subseteq}(\widetilde{G}, B)$ and $(\widetilde{G}, B)\widetilde{\subseteq}(\widetilde{F}, A)$.

Definition 8 ([3]). *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a universe U. "$(\widetilde{F}, A)$ AND $(\widetilde{G}, B)$", denoted by $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B)$, is defined by $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B) = (\widetilde{H}, A \times B)$, where $\widetilde{H}(\alpha, \beta) = \widetilde{F}(\alpha) \cap \widetilde{G}(\beta)$, for all $(\alpha, \beta) \in A \times B$.*

Definition 9 ([3]). *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a universe U. "$(\widetilde{F}, A)$ OR $(\widetilde{G}, B)$", denoted by $(\widetilde{F}, A)\widetilde{\vee}(\widetilde{G}, B)$, is defined by $(\widetilde{F}, A)\widetilde{\vee}(\widetilde{G}, B) = (\widetilde{H}, A \times B)$, where $\widetilde{H}(\alpha, \beta) = \widetilde{F}(\alpha) \cup \widetilde{G}(\beta)$, for all $(\alpha, \beta) \in A \times B$.*

Definition 10 ([3]). *The intersection of two intuitionistic fuzzy soft sets $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ over a universe U is an intuitionistic fuzzy soft set denoted by $(\widetilde{H}, C)$, where $C = A \cup B$ and*

$$\widetilde{H}(\alpha) = \begin{cases} \widetilde{F}(\alpha) & \textit{if } \alpha \in A - B, \\ \widetilde{G}(\alpha) & \textit{if } \alpha \in B - A, \\ \min\{\widetilde{F}(\alpha), \widetilde{G}(\alpha)\} & \textit{if } \alpha \in A \cap B, \end{cases}$$

for all $\alpha \in C$. This is denoted by $(\widetilde{H}, C) = (\widetilde{F}, A)\widetilde{\cap}(\widetilde{G}, B)$.

Definition 11 ([3]). *The union of two intuitionistic fuzzy soft sets $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ over a universe U is an intuitionistic fuzzy soft set denoted by $(\widetilde{H}, C)$, where $C = A \cup B$ and*

$$\widetilde{H}(\alpha) = \begin{cases} \widetilde{F}(\alpha) & \textit{if } \alpha \in A - B, \\ \widetilde{G}(\alpha) & \textit{if } \alpha \in B - A, \\ \max\{\widetilde{F}(\alpha), \widetilde{G}(\alpha)\} & \textit{if } \alpha \in A \cap B, \end{cases}$$

for all $\alpha \in C$. This is denoted by $(\widetilde{H}, C) = (\widetilde{F}, A)\widetilde{\cup}(\widetilde{G}, B)$.

In contrast with the above definitions of intuitionistic fuzzy soft set union and intersection, we may sometimes adopt different definitions of union and intersection given as follows.

Definition 12 ([3]). *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a universe U such that $A \cap B \neq \varnothing$. The bi-union of $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ is defined to be the intuitionistic fuzzy soft set $(\widetilde{H}, C)$, where $C = A \cap B$ and $\widetilde{H}(\alpha) = \widetilde{F}(\alpha) \cup \widetilde{G}(\alpha)$ for all $\alpha \in C$. This is denoted by $(\widetilde{H}, C) = (\widetilde{F}, A)\widetilde{\sqcup}(\widetilde{G}, B)$.*

Definition 13 ([3]). *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a universe U such that $A \cap B \neq \varnothing$.The bi-intersection $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ is defined to be the intuitionistic fuzzy soft set $(\widetilde{H}, C)$, where $C = A \cap B$ and $\widetilde{H}(\alpha) = \widetilde{F}(\alpha) \cap \widetilde{G}(\alpha)$ for all $\alpha \in C$. This is denoted by $(\widetilde{H}, C) = (\widetilde{F}, A)\widetilde{\sqcap}(\widetilde{G}, B)$.*

Definition 14 ([3])**.** *Let* $(\widetilde{F}, A)$ *and* $(\widetilde{G}, B)$ *be two intuitionistic soft sets over a universe* U*. The product of* $(\widetilde{F}, A)$ *and* $(\widetilde{G}, B)$ *is defined to be the intuitionistic soft set* $\left(\widetilde{F} \circ \widetilde{G}, C\right)$ *where* $C = A \cup B$ *and*

$$\mu_{(\widetilde{F} \circ \widetilde{G})(\alpha)}(x) = \begin{cases} \mu_{(\widetilde{F})(\alpha)}(x) & \text{if } \alpha \in A - B, \\ \mu_{(\widetilde{G})(\alpha)}(x) & \text{if } \alpha \in B - A, \\ \sup\limits_{x=ab} \min\left\{\mu_{(\widetilde{F})(\alpha)}(a), \mu_{(\widetilde{G})(\alpha)}(b)\right\} & \text{if } \alpha \in A \cap B, \end{cases}$$

and

$$\lambda_{(\widetilde{F} \circ \widetilde{G})(\alpha)}(x) = \begin{cases} \lambda_{(\widetilde{F})(\alpha)}(x) & \text{if } \alpha \in A - B, \\ \lambda_{(\widetilde{G})(\alpha)}(x) & \text{if } \alpha \in B - A, \\ \inf\limits_{x=ab} \max\left\{\lambda_{(\widetilde{F})(\alpha)}(a), \lambda_{(\widetilde{G})(\alpha)}(b)\right\} & \text{if } \alpha \in A \cap B, \end{cases}$$

for all $\alpha \in C$ *and* $x \in U$*. This is denoted by* $(\widetilde{F} \circ \widetilde{G}, C) = (\widetilde{F}, A) \circ (\widetilde{G}, B)$*.*

3. Intuitionistic fuzzy soft semi-ideals

Now, we are ready to give the definition of the intuitionistic fuzzy soft semi-ideal and examine it.

Definition 15. *An intuitionistic fuzzy set* $\left(\widetilde{F}, A\right)$ *over a semiring* S *is called an intuitionistic fuzzy soft semi-ideal over* S *if the following conditions are satisfied:*
(1) $\mu_{\widetilde{F}(\alpha)}(x-y) \geq \min\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{F}(\alpha)}(y)\right\}$ *and* $\lambda_{\widetilde{F}(\alpha)}(x-y) \leq \max\left\{\lambda_{\widetilde{F}(\alpha)}(x), \lambda_{\widetilde{F}(\alpha)}(y)\right\}$,
(2) $\mu_{\widetilde{F}(\alpha)}(xy) \geq \max\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{F}(\alpha)}(y)\right\}$ *and* $\lambda_{\widetilde{F}(\alpha)}(xy) \leq \min\left\{\lambda_{\widetilde{F}(\alpha)}(x), \lambda_{\widetilde{F}(\alpha)}(y)\right\}$
for all $x, y \in S$ *and* $\alpha \in A$*.*

Definition 16 ([10])**.** *For any intuitionistic fuzzy soft set* $A = \{\langle x, \mu_A(x), v_A(x)\rangle | \, x \in X\}$*,* $\Box A$ *and* $\Diamond A$ *are defined as follows:*
i) $\Box A = \{\langle x, \mu_A(x), \mu_A^c(x)\rangle | \, x \in X\}$;
ii) $\Diamond A = \{\langle x, v_A^c(x), v_A(x)\rangle | \, x \in X\}$.

Theorem 1. *Let* $\left(\widetilde{F}, A\right)$ *be an intuitionistic fuzzy soft semi-ideal over a semiring* S*. Then* $\left(\Box\widetilde{F}, A\right)$ *and* $\left(\Diamond\widetilde{F}, A\right)$ *are intuitionistic fuzzy soft semi-ideals over the semiring* S*.*

Proof. Since $\left(\widetilde{F}, A\right)$ is an intuitionistic fuzzy soft semi-ideal over a semiring S, for all $x, y \in S$ and $\alpha \in A$, we have

$\mu_{\widetilde{F}(\alpha)}(x-y) \geq \min\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{F}(\alpha)}(y)\right\}, \mu_{\widetilde{F}(\alpha)}(xy) \geq \max\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{F}(\alpha)}(y)\right\}$.
Then we obtain

$$\mu^{c}_{\widetilde{F}(\alpha)}(x-y) = 1 - \mu_{\widetilde{F}(\alpha)}(x-y)$$
$$\leq \max\left\{1 - \mu_{\widetilde{F}(\alpha)}(x), 1 - \mu_{\widetilde{F}(\alpha)}(y)\right\} = \max\left\{\mu^{c}_{\widetilde{F}(\alpha)}(x), \mu^{c}_{\widetilde{F}(\alpha)}(y)\right\},$$

and

$$\mu^{c}_{\widetilde{F}(\alpha)}(xy) = 1 - \mu_{\widetilde{F}(\alpha)}(xy)$$
$$\leq \min\left\{1 - \mu_{\widetilde{F}(\alpha)}(x), 1 - \mu_{\widetilde{F}(\alpha)}(y)\right\} = \min\left\{\mu^{c}_{\widetilde{F}(\alpha)}(x), \mu^{c}_{\widetilde{F}(\alpha)}(y)\right\}.$$

The check for $\left(\Diamond\widetilde{F}, A\right)$ is analogous to that for $\left(\Box\widetilde{F}, A\right)$.◄

Theorem 2. *Let $\left(\widetilde{F}, A\right)$ be an intuitionistic fuzzy soft semi-ideal over a semiring S. Then $\left(\widetilde{F}, A\right)$ is an intuitionistic fuzzy soft semi-ideal over S if and only if $\left(\widetilde{F}(\alpha)^{(r,t)}, A\right)$ is a soft semi-ideal of S for all $r \in (0,1]$, $t \in [1,0)$ and $\alpha \in A$.*

Proof. Let $\left(\widetilde{F}, A\right)$ be an intuitionistic fuzzy soft semi-ideal over a semiring S and $x, y \in \widetilde{F}(\alpha)^{(r,t)}$ and $s \in S$. Then $\mu_{\widetilde{F}(\alpha)}(x) \geq r$, $\mu_{\widetilde{F}(\alpha)}(y) \geq r$, $\lambda_{\widetilde{F}(\alpha)}(x) \leq t$ and $\lambda_{\widetilde{F}(\alpha)}(y) \leq t$. So, we have $\mu_{\widetilde{F}(\alpha)}(x-y) \geq \min\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{F}(\alpha)}(y)\right\} \geq r$ and $\lambda_{\widetilde{F}(\alpha)}(x-y) \leq \max\left\{\lambda_{\widetilde{F}(\alpha)}(x), \lambda_{\widetilde{F}(\alpha)}(y)\right\} \leq t$ by the assumption. Hence $x - y \in \widetilde{F}(\alpha)^{(r,t)}$. Also, $\mu_{\widetilde{F}(\alpha)}(sx) \geq \max\left\{\mu_{\widetilde{F}(\alpha)}(s), \mu_{\widetilde{F}(\alpha)}(x)\right\} \geq r$ and $\lambda_{\widetilde{F}(\alpha)}(sx) \leq \min\left\{\lambda_{\widetilde{F}(\alpha)}(s), \lambda_{\widetilde{F}(\alpha)}(x)\right\} \leq t$. Therefore $sx \in \widetilde{F}(\alpha)^{(r,t)}$ and so $\widetilde{F}(\alpha)^{(r,t)}$ is a semi-ideal of S. Conversely, let $\widetilde{F}(\alpha)^{(r,t)}$ be a semi-ideal of S. Then $x - y \in \widetilde{F}(\alpha)^{(r,t)}$ and $sx \in \widetilde{F}(\alpha)^{(r,t)}$ for all $x, y \in \widetilde{F}(\alpha)^{(r,t)}$ and $s \in S$. Suppose that $\mu_{\widetilde{F}(\alpha)}(x) = r_1$, $\mu_{\widetilde{F}(\alpha)}(y) = r_2$, $\min\{r_1, r_2\} = r_2$ and $\lambda_{\widetilde{F}(\alpha)}(x) = t_1$, $\lambda_{\widetilde{F}(\alpha)}(y) = t_2$, $\max\{t_1, t_2\} = t_2$. So, we can write $x - y \in \widetilde{F}(\alpha)^{(r_2,t_2)}$. Then $\mu_{\widetilde{F}(\alpha)}(x-y) \geq r_2 = \min\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{F}(\alpha)}(y)\right\}$ and $\lambda_{\widetilde{F}(\alpha)}(x-y) \leq t_2 = \max\left\{\lambda_{\widetilde{F}(\alpha)}(x), \lambda_{\widetilde{F}(\alpha)}(y)\right\}$. Also, in a similar way, $\mu_{\widetilde{F}(\alpha)}(sx) \geq \max\left\{\mu_{\widetilde{F}(\alpha)}(s), \mu_{\widetilde{F}(\alpha)}(x)\right\}$ and $\lambda_{\widetilde{F}(\alpha)}(sx) \leq \min\left\{\lambda_{\widetilde{F}(\alpha)}(s), \lambda_{\widetilde{F}(\alpha)}(x)\right\}$ is seen easily.◄

Theorem 3. *Let $\left(\widetilde{F}, A\right)$ and $\left(\widetilde{G}, B\right)$ be an intuitionistic fuzzy soft semi-ideal over a semiring S. Then so are $\left(\widetilde{F}, A\right) \widetilde{\wedge} \left(\widetilde{G}, B\right)$ and $\left(\widetilde{F}, A\right) \widetilde{\sqcap} \left(\widetilde{G}, B\right)$.*

Proof. We have $\left(\widetilde{F}, A\right) \widetilde{\wedge} \left(\widetilde{G}, B\right) = \left(\widetilde{H}, C\right)$, where $C = A \times B$ and $\widetilde{H}(\alpha, \beta) = \widetilde{F}(\alpha) \cap \widetilde{G}(\beta)$ for all $(\alpha, \beta) \in C$ from Definition 9. Since $\left(\widetilde{F}, A\right)$ and $\left(\widetilde{G}, B\right)$ are two intuitionistic fuzzy soft semi-ideals over S, for all $x, y \in S$ and $(\alpha, \beta) \in C$:

$$\begin{aligned}\mu_{\widetilde{H}(\alpha,\beta)}(x-y) &= \min\left\{\mu_{\widetilde{F}(\alpha)}(x-y), \mu_{\widetilde{G}(\beta)}(x-y)\right\} \geq \\ &\geq \min\left\{\min\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{F}(\alpha)}(y)\right\}, \min\left\{\mu_{\widetilde{G}(\beta)}(x), \mu_{\widetilde{G}(\beta)}(y)\right\}\right\} = \\ &= \min\left\{\mu_{\widetilde{H}(\alpha,\beta)}(x), \mu_{\widetilde{H}(\alpha,\beta)}(y)\right\},\end{aligned}$$

and

$$\begin{aligned}\lambda_{\widetilde{H}(\alpha,\beta)}(x-y) &= \max\left\{\lambda_{\widetilde{F}(\alpha)}(x-y), \lambda_{\widetilde{G}(\beta)}(x-y)\right\} \leq \\ &\leq \max\left\{\max\left\{\lambda_{\widetilde{F}(\alpha)}(x), \lambda_{\widetilde{F}(\alpha)}(y)\right\}, \max\left\{\lambda_{\widetilde{G}(\beta)}(x), \lambda_{\widetilde{G}(\beta)}(y)\right\}\right\} = \\ &= \max\left\{\lambda_{\widetilde{H}(\alpha,\beta)}(x), \lambda_{\widetilde{H}(\alpha,\beta)}(y)\right\}.\end{aligned}$$

$$\begin{aligned}\mu_{\widetilde{H}(\alpha,\beta)}(xy) &= \min\left\{\mu_{\widetilde{F}(\alpha)}(xy), \mu_{\widetilde{G}(\beta)}(xy)\right\} \geq \\ &\geq \min\left\{\max\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{F}(\alpha)}(y)\right\}, \max\left\{\mu_{\widetilde{G}(\beta)}(x), \mu_{\widetilde{G}(\beta)}(y)\right\}\right\} \geq \\ &\geq \max\left\{\min\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{G}(\beta)}(x)\right\}, \min\left\{\mu_{\widetilde{F}(\alpha)}(y), \mu_{\widetilde{G}(\beta)}(y)\right\}\right\} = \\ &= \max\left\{\mu_{\widetilde{H}(\alpha,\beta)}(x), \mu_{\widetilde{H}(\alpha,\beta)}(y)\right\},\end{aligned}$$

and

$$\begin{aligned}\lambda_{\widetilde{H}(\alpha,\beta)}(xy) &= \max\left\{\lambda_{\widetilde{F}(\alpha)}(xy), \lambda_{\widetilde{G}(\beta)}(xy)\right\} \leq \\ &\leq \max\left\{\min\left\{\lambda_{\widetilde{F}(\alpha)}(x), \lambda_{\widetilde{F}(\alpha)}(y)\right\}, \min\left\{\lambda_{\widetilde{G}(\beta)}(x), \lambda_{\widetilde{G}(\beta)}(y)\right\}\right\} \leq \\ &\leq \min\left\{\max\left\{\lambda_{\widetilde{F}(\alpha)}(x), \lambda_{\widetilde{G}(\beta)}(x)\right\}, \max\left\{\lambda_{\widetilde{F}(\alpha)}(y), \lambda_{\widetilde{G}(\beta)}(y)\right\}\right\} = \\ &= \min\left\{\lambda_{\widetilde{H}(\alpha,\beta)}(x), \lambda_{\widetilde{H}(\alpha,\beta)}(y)\right\}.\end{aligned}$$

Consequently, $\left(\widetilde{F}, A\right) \widetilde{\wedge} \left(\widetilde{G}, B\right)$ is an intuitionistic fuzzy soft semi-ideal over S. The proof of $\left(\widetilde{F}, A\right) \widetilde{\sqcap} \left(\widetilde{G}, B\right)$ is similar.◄

Theorem 4. *Let $\left(\widetilde{F}, A\right)$ and $\left(\widetilde{G}, B\right)$ be two intuitionistic fuzzy soft semi-ideals over a semiring S. If $\left(\widetilde{F}, A\right) \widetilde{\subseteq} \left(\widetilde{G}, B\right)$. Then so are $\left(\widetilde{F}, A\right) \widetilde{\vee} \left(\widetilde{G}, B\right)$ and $\left(\widetilde{F}, A\right) \widetilde{\sqcup} \left(\widetilde{G}, B\right)$.*

Proof. It is obvious.◄

Theorem 5. *Let $\left(\widetilde{F}, A\right)$ and $\left(\widetilde{G}, B\right)$ be two intuitionistic fuzzy soft semi-ideals over a semiring S. Then so are $\left(\widetilde{F}, A\right) \widetilde{\cap} \left(\widetilde{G}, B\right)$ and $\left(\widetilde{F}, A\right) \widetilde{\cup} \left(\widetilde{G}, B\right)$.*

Proof. For any $x, y \in S$ and $\alpha \in C$, we consider the following cases:

(1) Let $\alpha \in A \diagdown B$. Then we have

$$\mu_{\widetilde{H}(\alpha)}(x-y) = \mu_{\widetilde{F}(\alpha)}(x-y) \geq \min\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{F}(\alpha)}(y)\right\} = \min\left\{\mu_{\widetilde{H}(\alpha)}(x), \mu_{\widetilde{H}(\alpha)}(y)\right\},$$

$$\lambda_{\widetilde{H}(\alpha)}(x-y) = \lambda_{\widetilde{F}(\alpha)}(x-y) \leq \max\left\{\lambda_{\widetilde{F}(\alpha)}(x), \lambda_{\widetilde{F}(\alpha)}(y)\right\} = \max\left\{\lambda_{\widetilde{H}(\alpha)}(x), \lambda_{\widetilde{H}(\alpha)}(y)\right\},$$

and

$$\mu_{\widetilde{H}(\alpha)}(xy) = \mu_{\widetilde{F}(\alpha)}(xy) \geq \max\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{F}(\alpha)}(y)\right\} = \max\left\{\mu_{\widetilde{H}(\alpha)}(x), \mu_{\widetilde{H}(\alpha)}(y)\right\},$$

$$\lambda_{\widetilde{H}(\alpha)}(xy) = \lambda_{\widetilde{F}(\alpha)}(xy) \leq \min\left\{\lambda_{\widetilde{F}(\alpha)}(x), \lambda_{\widetilde{F}(\alpha)}(y)\right\} = \min\left\{\lambda_{\widetilde{H}(\alpha)}(x), \lambda_{\widetilde{H}(\alpha)}(y)\right\}.$$

(2) Let $\alpha \in B \diagdown A$. This case is similar to (1).

(3) Let $\alpha \in A \cap B$. Then $\widetilde{H}(\alpha) = \widetilde{F}(\alpha) \cap \widetilde{G}(\alpha)$. It follows that

$$\begin{aligned}
\mu_{\widetilde{H}(\alpha)}(x-y) = \min\left\{\mu_{\widetilde{F}(\alpha)}(x-y), \mu_{\widetilde{G}(\alpha)}(x-y)\right\} \geq \\
&\geq \min\left\{\min\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{F}(\alpha)}(y)\right\}, \min\left\{\mu_{\widetilde{G}(\alpha)}(x), \mu_{\widetilde{G}(\alpha)}(y)\right\}\right\} = \\
&= \min\left\{\min\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{G}(\alpha)}(x)\right\}, \min\left\{\mu_{\widetilde{F}(\alpha)}(y), \mu_{\widetilde{G}(\alpha)}(y)\right\}\right\} = \\
&= \min\left\{\mu_{\widetilde{H}(\alpha)}(x), \mu_{\widetilde{H}(\alpha)}(y)\right\}
\end{aligned}$$

and

$$\begin{aligned}
\lambda_{\widetilde{H}(\alpha)}(x-y) = \max\left\{\lambda_{\widetilde{F}(\alpha)}(x-y), \lambda_{\widetilde{G}(\alpha)}(x-y)\right\} \leq \\
&\leq \max\left\{\max\left\{\lambda_{\widetilde{F}(\alpha)}(x), \lambda_{\widetilde{F}(\alpha)}(y)\right\}, \max\left\{\lambda_{\widetilde{G}(\alpha)}(x), \lambda_{\widetilde{G}(\alpha)}(y)\right\}\right\} = \\
&= \max\left\{\max\left\{\lambda_{\widetilde{F}(\alpha)}(x), \lambda_{\widetilde{G}(\alpha)}(x)\right\}, \max\left\{\lambda_{\widetilde{F}(\alpha)}(y), \lambda_{\widetilde{G}(\alpha)}(y)\right\}\right\} = \\
&= \max\left\{\lambda_{\widetilde{H}(\alpha)}(x), \lambda_{\widetilde{H}(\alpha)}(y)\right\}.
\end{aligned}$$

$\mu_{\widetilde{H}(\alpha)}(xy) \geq \max\left\{\mu_{\widetilde{H}(\alpha)}(x), \mu_{\widetilde{H}(\alpha)}(y)\right\}$ and $\lambda_{\widetilde{H}(\alpha)}(xy) \leq \min\left\{\lambda_{\widetilde{H}(\alpha)}(x), \lambda_{\widetilde{H}(\alpha)}(y)\right\}$, are similarly proved.

Consequently, $\left(\widetilde{F}, A\right) \widetilde{\cap} \left(\widetilde{G}, B\right)$ is an intuitionistic fuzzy soft semi-ideal over a semiring S. $\left(\widetilde{F}, A\right) \widetilde{\cup} \left(\widetilde{G}, B\right)$ can be similarly proved as well.◄

Theorem 6. *Let* $\left(\widetilde{F}, A\right)$ *and* $\left(\widetilde{G}, B\right)$ *be two intuitionistic fuzzy soft semi-ideals over a semiring* S. *Then so is* $\left(\widetilde{F}, A\right) \circ \left(\widetilde{G}, B\right)$.

Proof. For any $x, y \in S$ and $\alpha \in A \cup B$, we consider the following cases:

(1) Let $\alpha \in A \diagdown B$. Then we get

$$\mu_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(x-y) = \mu_{\widetilde{F}(\alpha)}(x-y) \geq$$

$$\geq \min\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{F}(\alpha)}(y)\right\} = \min\left\{\mu_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(x), \mu_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(y)\right\},$$

$$\lambda_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(x-y) = \lambda_{\widetilde{F}(\alpha)}(x-y) \leq$$

$$\leq \max\left\{\lambda_{\widetilde{F}(\alpha)}(x), \lambda_{\widetilde{F}(\alpha)}(y)\right\} = \max\left\{\lambda_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(x), \lambda_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(y)\right\},$$

and

$$\mu_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(xy) = \mu_{\widetilde{F}(\alpha)}(xy) \geq$$
$$\geq \max\left\{\mu_{\widetilde{F}(\alpha)}(x), \mu_{\widetilde{F}(\alpha)}(y)\right\} = \max\left\{\mu_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(x), \mu_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(y)\right\},$$
$$\lambda_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(xy) = \lambda_{\widetilde{F}(\alpha)}(xy) \leq$$
$$\leq \min\left\{\lambda_{\widetilde{F}(\alpha)}(x), \lambda_{\widetilde{F}(\alpha)}(y)\right\} = \min\left\{\lambda_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(x), \lambda_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(y)\right\}.$$

(2) Let $\alpha \in B \diagdown A$. This case is similar to (1).

(3) Let $\alpha \in A \cap B$. Then we obtain

$$\begin{aligned}\mu_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(x) &= \sup_{x=x_1x_2} \min\left\{\mu_{\widetilde{F}(\alpha)}(x_1), \mu_{\widetilde{G}(\alpha)}(x_2)\right\} \leq \\ &\leq \sup_{xy=x_1x_2y} \min\left\{\mu_{\widetilde{F}(\alpha)}(x_1y), \mu_{\widetilde{G}(\alpha)}(x_2y)\right\} \leq \\ &\leq \sup_{xy=zt} \min\left\{\mu_{\widetilde{F}(\alpha)}(z), \mu_{\widetilde{G}(\alpha)}(t)\right\} = \\ &= \mu_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(xy).\end{aligned}$$

Similarly, we can write $\mu_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(y) \leq \mu_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(xy)$. Therefore, $\mu_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(xy) \geq \max\left\{\mu_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(x), \mu_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(y)\right\}$.

Also,

$$\begin{aligned}\lambda_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(x) &= \inf_{x=x_1x_2} \max\left\{\lambda_{\widetilde{F}(\alpha)}(x_1), \lambda_{\widetilde{G}(\alpha)}(x_2)\right\} \geq \\ &\geq \inf_{xy=x_1x_2y} \max\left\{\lambda_{\widetilde{F}(\alpha)}(x_1y), \lambda_{\widetilde{G}(\alpha)}(x_2y)\right\} \geq \\ &\geq \inf_{xy=zt} \max\left\{\lambda_{\widetilde{F}(\alpha)}(z), \lambda_{\widetilde{G}(\alpha)}(t)\right\} = \\ &= \lambda_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(xy).\end{aligned}$$

Similarly, we can write $\lambda_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(y) \geq \lambda_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(xy)$. Hence, $\lambda_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(xy) \leq \min\left\{\lambda_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(x), \lambda_{\left(\widetilde{F}\circ\widetilde{G}\right)(\alpha)}(y)\right\}$. Then the proof is completed.◄

Theorem 7. *Let S and S' be two soft semirings and $f : S \to S'$ be an homomorphism.*
a) If f is onto and $\left(\widetilde{F}, A\right)$ is an intuitionistic fuzzy soft semi-ideal over S, then $f\left(\left(\widetilde{F}, A\right)\right)$ is an intuitionistic fuzzy soft semi-ideal over S'.
b) If $\left(\widetilde{G}, B\right)$ is an intuitionistic fuzzy soft semi-ideal over S', then $f^{-1}\left(\left(\widetilde{G}, B\right)\right)$ is an intuitionistic fuzzy soft semi-ideal over S.

Proof. Trivial.◄

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