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On Building Two-Player Games with Treatment Schedules for the SIR Model

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Abstract. In this study, a treatment argument is provided as a discrete two-player game related to an epidemiological dynamics, so-called, Susceptible- Infectious-Recovered (SIR) model. Here, a simple discrete version of the dynamics of SIR model is considered within a treatment structure in such a way to control the behaviour of each candidate: population of the susceptible, infected and recovered people, respectively. In this regard, several two-player game models are proposed, where one player follows its own existed policy where as the other tries to track its opponent's treatment schedule as close as possible. In this regard, different strategies are built for one player to catch the other in a two-player game environment, where one player determines the total number of susceptible or infected people at a given period, in the meantime, the other tries to build its corresponding treatment policy to get closer to its opponent's counting schedule. The main contribution of this work is to build a better treatment schedule by using a game theoretical point of view to cure the population suffered from an infectious disease. At the end, the work is related to pursuer-evasion discrete games and the idea could be implemented on compartmental models like COVID-19 and transportation problems.

Key Words and Phrases: two-player game, SIR model, pursuit-evasion game, treatment schedule.

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1. Introduction

Recently, there has been a great tendency for utilizing mathematical modelling, most of which are derived from continuous environments for real life problems in different fields of science and technology. Among them, computational epidemiology provides different tools and methodologies for modelling the spread of infectious diseases with possible various treatments, such as the use of vaccination [5, 6, 7, 10], hand washing [1, 42], social distancing [40], having related

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medications, drugs [3], etc. In this regard, game theory stands forward to model different scenarios to decide which is better or the best in the face of preventing the deadly disease before it is too late. For that analysis, an appropriate decision making process plays a crucial role in a given competitive environment, where one player's decision would automatically influence the outcome of a situation for all players involved. Here, the players could be seen as individuals, groups of people, treatments or computer programs, aiming to develop strategies to optimize the gain, so-called pay-off. In literature, we refer to [26, 28, 29, 33, 34], and [12, 14, 15, 17, 37] for building game theoretical methodologies in epidemiology.

In addition, we refer to the book [34] for an introduction of modelling and solving two-player and pursuit-evasion games with both discrete and continuous point of views. Moreover, we refer to the paper [33] which examines two-player zero-sum differential game known as the target guarding problem. Furthermore, we refer to [12] for an overview of game theoretical perspectives in the dynamics of various sort of infectious disease models with latest developments. Besides, we refer to the paper [41] for stochastic differential games in mathematical economy. Apart from that, we refer to [16, 38, 39] for modelling gene-environment networks and [9, 13, 19, 44] for further analysis of different dynamical models with operation research point of views.

In the context of two-player games in transportation and queueing theory, we refer to [18] for constructing two game models as Nash non-cooperative and Stackelberg games with additional problems in transportation systems modelling by using decision-making models for planning and operating transportation systems. Moreover, we refer to the papers [20, 21, 22, 23, 24, 25] for modelling certain class of transportation problems in queueing theory with statistical and probabilistic aspects. Here, it is worth to note that those problems and approaches are motivating to design two-player game environments in which one player, a leader, knows or tracks the path of the other player, a follower, who is certainly capable of taking any decision in this regard.

Recently, there has been a growing interest toward building effective and control based environments for preventing the spread of an infectious disease, such as influenza [11, 17, 37], smallpox [8], chickenpox [36], and more recently COVID-19 [31, 43]. For that reason, new mathematical models have been constructed to investigate and determine different dynamical behaviour of the widely spread diseases [1, 31, 43]. Here, we refer to the recent papers [1, 2, 3, 40, 42, 43] for modelling and analysing COVID-19 and its evolution process in different fields of the subject under certain treatment measures. Moreover, we refer to the book [4] for examining different models of infectious diseases of humans from mathematical point of view (see also [35]).

In this paper, we design two-player games, where each player's position is determined by the system emerged from dynamics of the SIR model [4] with possible treatments posed on the system. In this regard, one player is allowed to follow any possible treatments, where as the other tries to catch its opponent treatment policy as near as possible. Here, we define one player's position in three different ways, namely, as the current number of susceptible, infected and recovered population, respectively. More precisely, we propose three kinds of two-player game models in which each player's position is respectively related to the number of these three categories; susceptible, infectious and recovered population at a given period of time. Moreover, we portray a different playing ground where one player which follows its own policy is considered as the evader, where as the other which tracks its opponent treatment model is regarded as the pursuer. The game is started by the evader which is followed by the pursuer, each player moves one step at each period and the game finishes whenever the evader is caught by the pursuer within a small neighbourhood. For such game models, we refer to [26, 28, 29, 31] for examining two-player discrete, games where the position of each player is formulated by controls taken from finite sets.

The paper is organized as follows. In Section 2, we provide the statement of the problem by considering the dynamics of the SIR model and its discrete version by adopting Euler discretization technique [26, 27, 32]. Moreover, we propose a treatment policy for the discrete model of the SIR and build a proper playing ground for interactions of two players, each of whom consists of different treatment strategies for overcoming the disease. In Section 3, we design three kinds of two-player games, where each player's position is determined by the discrete SIR model with possible treatment policies. In Section 4, the paper concludes with some remarks and future works related to the topic.

2. Statement of the Problem

In this part, we consider the following dynamics of an epidemic [4]:

$$\begin{cases} S'(t) = -\alpha S(t)I(t), \\ I'(t) = \alpha S(t)I(t) - \beta I(t), \\ R'(t) = \beta I(t), \end{cases} \tag{1}$$

where constants $\alpha > 0$, $\beta > 0$ represent interaction rates of the disease and $S(t), I(t), R(t)$ stands for the existence population of susceptible, infected and recovered people at time t, respectively. For having unique solution, we state previously that $S(0) = S_0$, initial population, $I(0) = I_0 > 0$, initial cases of disease and number of initial recovered patients $R(0) = R_0$, mostly assumed to be zero at the beginning of the outbreak. Since the dynamics of the system (1) is designed as having no death cases, or contributing so less comparing to the general population due to the disease, the total number of population stays fixed. More precisely, it is well seen from the system that $S(t)+I(t)+R(t) = S_0+I_0+R_0$.

Consider the following discrete form:

$$\begin{cases} S(n+1) = S(n) - h\alpha S(n)I(n), \\ I(n+1) = I(n) + h(\alpha S(n)I(n) - \beta I(n)), \end{cases} \tag{2}$$

where $h > 0$ is a fixed step size close to zero with $S(0) = S_0$, $I(0) = I_0 > 0$. After the spread of a disease, the number of recovered patients at m^{th} period, could be seen in the following way:

$$R(m) = N - S(m) - I(m), \tag{3}$$

where N represents the total number of population. It is well noted that at the beginning of the spread of a disease, we can assume without loss of generality that $S_0 \approx N$, and $I_0 \approx 0$. Here, the system (2) can be considered in the following way:

$$\begin{cases} S(n+1) = S(n) - u(n), \\ I(n+1) = I(n) + v(n), \end{cases} \tag{4}$$

where $u(n), v(n) > 0$ for all $n \in \mathbb{N}$. Here, $u(n) := h\alpha S(n)I(n)$ and $v(n) := h(\alpha S(n)I(n) - \beta I(n))$. Moreover, we have the following interaction:

$$u(n) - v(n) = h\beta I(n) > 0, \quad n \geq 1. \tag{5}$$

In addition, one can note that each $k \in \mathbb{N}$ for $S(k) > 0$, the sequence of susceptible population $\{S(0), S(1), \cdots, S(k)\}$ is decreasing, whereas the sequence of infected population $\{I(0), I(1), \cdots, I(k)\}$ is increasing as days go by. Moreover, total number of susceptible and infected individuals at $(k+1)^{th}$ period would be

$$S(k+1) \approx N - \sum_{i=0}^{k} u(i) \quad \text{and} \quad I(k+1) \approx \sum_{i=0}^{k} v(i), \tag{6}$$

respectively. Hence, the total number of recovered patients at $(k+1)^{th}$ period becomes

$$R(k+1) \approx \sum_{i=0}^{k} (u(i) - v(i)). \tag{7}$$

Now, we are ready to design a treatment model posed on the discrete system. For this reason, we consider the following model:

$$\begin{cases} S(n+1) = S(n) - u(n) + \mu u(n), \\ I(n+1) = I(n) + v(n) - \eta v(n), \end{cases} \tag{8}$$

where $\mu, \eta \in [0,1]$, real numbers corresponding to the effectiveness rate of the treatment delivered. For $\eta = \mu = 0$, we obtain the model (4) which means no treatment is involved. On the other hand, if $\eta = \mu = 1$, we have the full treatment for the disease, such as the use of vaccine in the treatment, which means the epidemic ends instantly, i.e., $S(n) = N$ and $I(n) = 0$, for each $n \in \mathbb{N}$. Hence, one can see that the effectiveness of the treatment becomes apparent whenever μ, η are close to one. The next section presents the study of that treatment model which determines each position of the players in a constructed two-player game.

3. Constructing Two-Player Games in the SIR Model with Possible Treatment Policies

In this section, we design two-player games, where each player's position is determined by the system (8) with different $\mu, \eta \in [0,1]$, where one player builds its own treatment policy, whereas the other tries to catch its opponent's policy as close as possible. In this game, each player moves one step at a time and the game finishes whenever one player (the evader) is trapped by the other (the pursuer) within a small neighbourhood, i.e., the number of susceptible, infected and recovered population in each policy becomes respectively closer to each other. Hence, we consider three different game models, simultaneously investigating interactions of each player's positions related to current number of susceptible, infected and recovered population. In this game, we concentrate on the following discrete model (8) which determines the move of each player:

$$\underbrace{\begin{cases} S_e(n+1) = S_e(n) - (1-\mu_e)u_e(n), \\ I_e(n+1) = I_e(n) + (1-\eta_e)v_e(n), \end{cases}}_{\text{The evader's model}} \qquad \underbrace{\begin{cases} S_p(n+1) = S_p(n) - (1-\mu_p)u_p(n), \\ I_p(n+1) = I_p(n) + (1-\eta_p)v_p(n), \end{cases}}_{\text{The pursuer's model}}$$

where $S_e(0) = S_e, S_p(0) = S_p$ with $I_e(0) = I_e > 0$ and $I_p(0) = I_p > 0$, both of them are close to zero, possibly different from each other. More precisely, without loss of generality, it is assumed that $S_e, S_p \approx N$, and $I_e, I_p \approx 0$, where N stands for the total number of people in the region, each of whom is capable of getting sick from an infection during this period. Here, $S_e(k)$ and $I_e(k)$ represent the total number of susceptible and infected people at the period k for the evader, respectively. Similarly, $S_p(k)$ and $I_e(k)$ stand for the total number of susceptible and infected people at the period k for the pursuer, respectively. For recovered population, we use the formula (3) to determine the positions of each player.

Here, we call (μ_e, η_e) as the treatment policy for the evader and similarly (μ_p, η_p) for the pursuer. Moreover, a position of the evader at k^{th} step is seen as $S_e(k)$ if the chasing is built on the number of susceptible people or it is taken as $I_e(k)$ if the chasing is constructed on the number of infected population. Similarly, a position of the pursuer at k^{th} step would be considered as $S_p(k)$, $I_p(k)$ or $R_p(k)$ with the policy (μ_p, η_p) depending on the game structure. For being precise, we define the two-player game structure related to the positions of each player determined by susceptible, infected and recovered population at each period of time as the first, second and third game model, respectively.

Theorem 1. *In a two-player first game model, for any treatment policy of the evader, (μ_e, η_e) with initial environment (S_e, I_e), there exists a treatment policy (μ_p, η_p) with initial environment (S_p, I_p) for the pursuer to catch its opponent within a small neighbourhood. Moreover, there are many treatment policies favouring the pursuer in this regard.*

Proof. Assume that the evader follows the policy (μ_e, η_e) with initial environment (S_e, I_e), i.e., $S_e \approx N$ and $I_e(0) = I_e > 0$. Without loss of generality, initially, I_e is supposed to be close to zero. Let $\rho \geq 1$, any positive real number. Here, the sequences

$$(u_e(0), u_e(1), \cdots, u_e(k)) \quad \text{and} \quad (v_e(0), v_e(1), \cdots, v_e(k)), \tag{9}$$

determine the position of the evader at $(k+1)^{th}$ level, i.e., $(S_e(k+1)$ with $I_e(k+1)$ in our case. Now, for each ρ, we develop the following chasing strategies for the pursuer:

$$\mu_p = \frac{\rho - 1 + \mu_e}{\rho} \quad \text{and} \quad \eta_p = \eta_e, \tag{10}$$

with

$$S_p \approx S_e \quad \text{and} \quad I_p = \rho I_e, \tag{11}$$

Under (10) and (11), we build the sequences

$$(u_p(0), u_p(1), \cdots, u_p(k)) \quad \text{and} \quad (v_p(0), v_p(1), \cdots, v_p(k)), \tag{12}$$

set up the position of the pursuer at $(k+1)^{th}$ period, namely, $(S_p(k+1)$ with $I_p(k+1))$. Here, we show that for each $k \in \mathbb{Z}^+$, the relations

$$S_e(k) \approx S_p(k) \quad \text{and} \quad I_p(k) \approx \rho I_e(k) \tag{13}$$

hold, i.e., the pursuer traps the evader. For the proof of (13), we use induction. Namely, we check first whether both case holds for $k = 1$. We have

$$S_p(1) - S_e(1) \approx (S_p - S_e) + h\alpha(1-\mu_e)(\frac{1}{\rho}S_p\rho I_e - S_e I_e) \approx 0. \tag{14}$$

Similarly, we obtain $I_p(1) \approx \rho I_e(1)$. From (14), the case for $k = 1$ holds. Now, assume that the case $k = (n-1)$ holds, we prove that the case $k = n$ satisfies. From induction hypothesis, $S_p(n-1) \approx S_e(n-1)$ and $I_p(n-1) \approx \rho I_e(n-1)$. We have

$$S_p(n) - S_e(n) \approx h\alpha(1-\mu_e)\Big(\frac{1}{\rho}S_p(n-1)I_p(n-1) - S_e(n-1)I_e(n-1)\Big) \approx 0.$$

Hence, $S_p(n) \approx S_e(n)$ for each $n \in \mathbb{N}$. Similarly, one can have $I_p(n) \approx \rho I_e(n)$ for each $n \in \mathbb{N}$.

As a result, for any $\rho \geq 1$, we prove that the treatment policy (μ_p, η_p) satisfying (10) and (11) builds different chasing strategies in favour of the pursuer to catch its opponent in any period which proves the theorem. ◄

Remark 1. *In Theorem 1, we see that there are many strategies favouring the pursuer to trap the evader within some neighbourhood at any period of time. For any given $k \in \mathbb{N}$ and $\epsilon > 0$, initial environment S_p for the pursuer could be chosen as close to S_e to make the gap stays within ϵ neighbourhood*

$$|S_e(k) - S_p(k)| < \epsilon.$$

In Theorem 1, one can see that for $\rho \approx 1$, we obtain the same scenario for the second game model in which any treatment policy that the evader follows, one can design a tracking schedule for the pursuer to get closer to its opponent. Hence, we have the following result as an immediate consequence of Theorem 1.

Corollary 1. *In a two-player second game model, for any treatment policy of the evader, (μ_e, η_e) with initial environment (S_e, I_e), there exists different treatment policies (μ_p, η_p) with initial environment (S_p, I_p) in favour of the pursuer to trap its opponent.*

Remark 2. *With Theorem 1 and Corollary 1, we see that it is possible to design chasing strategies for two-player game in regard of different type of treatment models as (8). Moreover, one can also build different chasing strategies in a two-player third game model, where the position of each player is determined by the number of recovered patients.*

4. Conclusion

In this paper, we establish two-player game models with treatments for the discrete SIR model. In this regard, we concentrated on the discrete models emerged from the continuous dynamics of the SIR model which motivates us to create a discrete playground for some continuous models like a spread model of COVID-19 [2, 3] and any other infectious diseases. Hopefully, some discretization techniques applied for those continuous models could provide us discrete models like (8) (see e.g., [26, 30, 31, 32]) which creates a possible playing environment for designing corresponding two-player games controlled by possible treatment policies.

As a future work, it would be an interesting attempt to investigate whether there exists a playing environment for two-player games in regard of the following general discrete model:

$$\begin{cases} S_\lambda(n+1) = \lambda S_\lambda(n) - h\alpha S_\lambda(n) I_\lambda(n), \\ I_\lambda(n+1) = \lambda I_\lambda(n) + h(\alpha S_\lambda(n) I_\lambda(n) - \beta I_\lambda(n)), \end{cases} \tag{15}$$

where $\lambda > 1$. Of course, to have a well-defined structure, initial environments $I_\lambda(0)$ and $S_\lambda(0)$ ($h > 0$ is also included if it is necessary) should be chosen in such a way that the sequence $\{S_\lambda(0), S_\lambda(1), \cdots, S_\lambda(k)\}$ becomes a decreasing finite sequence, whereas the sequence $\{I_\lambda(0), I_\lambda(1), \cdots, I_\lambda(k)\}$ becomes an increasing finite sequence for each $k \in \mathbb{N}$ satisfying $S_\lambda(k) > 0$. Here, it is worth to mention that the main difference between the model (15) and (2) is that spread of the disease in (15) is more effective than the one in (2), i.e., the above epidemic model is more contagious than the other. For answering that question, we may utilize some topological properties of the following set [30]:

$$X_m(\lambda) := \left\{ \alpha_0 + \alpha_1 \lambda + ... + \alpha_k \lambda^k : |\alpha_i| \leq m, k \in \mathbb{N} \right\},$$

for understanding the behaviour of each consecutive term [30]. Those properties could help us to get more familiar with the playing ground of each player and their interaction among them.

In addition, it could be an interesting and motivating attempt to construct two-player game models in transportation. More precisely, one could stay focused on mathematical models of moving particles [20, 21, 22, 23] by building discrete playing environments with players considered as moving particles controlled by finite sets.

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