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A New Theorem on Generalized Absolute Matrix Summability

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Abstract. In this paper, a general theorem dealing with $\varphi - |A, p_n|_k$ summability of an infinite series has been proved by using an almost increasing sequence. Also, some results have been obtained.

Key Words and Phrases: absolute matrix summability, almost increasing sequences, infinite series, Hölder's inequality, Minkowski's inequality, Riesz mean, summability factor.

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1. Introduction

A positive sequence (b_n) is said to be almost increasing if there exist a positive increasing sequence (c_n) and two positive constants K and L such that $Kc_n \leq b_n \leq Lc_n$ (see [1]). Obviously, every increasing sequence is almost increasing. However, the converse need not be true as can be seen by taking an example, say, $b_n = ne^{(-1)^n}$. For any sequence (λ_n) we write $\Delta\lambda_n = \lambda_n - \lambda_{n+1}$. Let $\sum a_n$ be a given infinite series with the partial sums (s_n). Let (p_n) be a sequence of positive numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty \quad as \quad (n \to \infty), \quad (P_{-i} = p_{-i} = 0, \quad i \geq 1).$$

The sequence-to-sequence transformation

$$\sigma_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v$$

defines the sequence (σ_n) of the $\left(\bar{N}, p_n\right)$ mean of the sequence (s_n), generated by the sequence of coefficients (p_n) (see [10]). The series $\sum a_n$ is said to be summable $\left|\bar{N}, p_n\right|_k$, $k \geq 1$, if (see [2])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} |\sigma_n - \sigma_{n-1}|^k < \infty.$$

Let $A = (a_{nv})$ be a normal matrix, i.e., a lower triangular matrix of nonzero diagonal entries. Then A defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $As = (A_n(s))$, where

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v, \quad n = 0, 1, ...$$

Let (φ_n) be any sequence of positive real numbers. The series $\sum a_n$ is said to be summable $\varphi - |A, p_n|_k$, $k \geq 1$, if (see [16])

$$\sum_{n=1}^{\infty} \varphi_n^{k-1} |A_n(s) - A_{n-1}(s)|^k < \infty.$$

If we take $\varphi_n = \frac{P_n}{p_n}$, then $\varphi - |A, p_n|_k$ summability reduces to $|A, p_n|_k$ summability (see [25]). Also, if we take $\varphi_n = \frac{P_n}{p_n}$ and $a_{nv} = \frac{p_v}{P_n}$, then we get $|\bar{N}, p_n|_k$ summability. Furthermore, if we take $\varphi_n = n$, $a_{nv} = \frac{p_v}{P_n}$ and $p_n = 1$ for all values of n, then $\varphi - |A, p_n|_k$ summability reduces to $|C, 1|_k$ summability (see [9]). Given a normal matrix $A = (a_{nv})$, two lower semimatrices $\bar{A} = (\bar{a}_{nv})$ and $\hat{A} = (\hat{a}_{nv})$ are defined as follows:

$$\bar{a}_{nv} = \sum_{i=v}^{n} a_{ni}, \quad n, v = 0, 1, ... \tag{1}$$

$$\hat{a}_{00} = \bar{a}_{00} = a_{00}, \quad \hat{a}_{nv} = \bar{a}_{nv} - \bar{a}_{n-1,v}, \quad n = 1, 2, ... \tag{2}$$

and

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v = \sum_{i=0}^{n} \bar{a}_{ni} a_i \tag{3}$$

$$\bar{\Delta} A_n(s) = \sum_{i=0}^{n} \hat{a}_{ni} a_i. \tag{4}$$

2. Known Result

In [8], Bor has proved the following theorem for $\left|\bar{N}, p_n\right|_k$ summability factors of an infinite series. For more studies on Riesz summability of infinite series, we can refer to [3, 4, 5, 6, 7].

Theorem 1. *Let (X_n) be an almost increasing sequence and let there be sequences (β_n), (λ_n) such that*

$$|\Delta\lambda_n| \leq \beta_n, \tag{5}$$

$$\beta_n \to 0 \quad as \quad n \to \infty, \tag{6}$$

$$\sum_{n=1}^{\infty} n\,|\Delta\beta_n|\,X_n < \infty, \tag{7}$$

$$|\lambda_n| X_n = O(1). \tag{8}$$

If

$$\sum_{n=1}^{m} \frac{|\lambda_n|}{n} = O(1) \quad as \quad m \to \infty, \tag{9}$$

$$\sum_{n=1}^{m} \frac{1}{n}|t_n|^k = O(X_m) \quad as \quad m \to \infty \tag{10}$$

and (p_n) is a sequence such that

$$\sum_{n=1}^{m} \frac{p_n}{P_n}\,|t_n|^k = O\left(X_m\right) \quad as \quad m \to \infty, \tag{11}$$

where (t_n) is the nth $(C,1)$ mean of the sequence (na_n), then the series $\sum a_n\lambda_n$ is summable $\left|\bar{N}, p_n\right|_k, k \geq 1$.

3. Main Result

Many studies have been done for absolute matrix summability methods of an infinite series (see [12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24]). The aim of this paper is to generalize Theorem 1 for absolute matrix summability. Now we shall prove the following theorem.

Theorem 2. *Let $A=(a_{nv})$ be a positive normal matrix such that*

$$\overline{a}_{n0}=1,\ n=0,1,..., \tag{12}$$

$$a_{n-1,v}\geq a_{nv}\ for\ n\geq v+1, \tag{13}$$

$$a_{nn}=O\left(\frac{p_n}{P_n}\right). \tag{14}$$

Let (X_n) be an almost increasing sequence and $(\frac{\varphi_n p_n}{P_n})$ be a non-increasing sequence. If conditions (5)-(9) of Theorem 1 and

$$\sum_{n=1}^{m}\varphi_n^{k-1}\left(\frac{p_n}{P_n}\right)^{k-1}\frac{1}{n}|t_n|^k=O(X_m)\quad as\quad m\to\infty, \tag{15}$$

$$\sum_{n=1}^{m}\varphi_n^{k-1}\left(\frac{p_n}{P_n}\right)^{k}|t_n|^k=O(X_m)\quad as\quad m\to\infty \tag{16}$$

are satisfied, then the series $\sum a_n\lambda_n$ is summable $\varphi-|A,p_n|_k$, $k\geq 1$.

$\varphi-|A,p_n|_k$ summability method is more general than $\left|\bar{N},p_n\right|_k$ summability method. By using a positive normal matrix and some suitable conditions, Theorem 2 on absolute matrix summability method is obtained. This indicates the importance of the theorem. If we take $\varphi_n=\frac{P_n}{p_n}$ and $a_{nv}=\frac{p_v}{P_n}$ in Theorem 2, then we get Theorem 1. In this case, the conditions (15) and (16) reduce to the conditions (10) and (11), respectively. Also, the condition "$\left(\frac{\varphi_n p_n}{P_n}\right)$ is a non-increasing sequence" and the conditions (12)-(14) are automatically satisfied.

We need the following lemma for the proof of Theorem 2.

Lemma 1. *([11]) Under the conditions on (X_n), (β_n) and (λ_n) as taken in the statement of Theorem 2, the following conditions hold:*

$$n\beta_nX_n=O\left(1\right)\quad as\quad n\to\infty, \tag{17}$$

$$\sum_{n=1}^{\infty}\beta_nX_n<\infty. \tag{18}$$

4. Proof of Theorem 2

Let (I_n) denote A-transform of the series $\sum a_n\lambda_n$. Then, by (3) and (4), we have

$$\bar{\Delta} I_n = \sum_{v=0}^{n} \hat{a}_{nv} a_v \lambda_v = \sum_{v=1}^{n} \frac{\hat{a}_{nv}\lambda_v}{v} v a_v.$$

Applying Abel's transformation to this sum, we get

$$\begin{aligned}
\bar{\Delta} I_n &= \sum_{v=1}^{n-1} \Delta_v \left(\frac{\hat{a}_{nv}\lambda_v}{v}\right) \sum_{r=1}^{v} r a_r + \frac{\hat{a}_{nn}\lambda_n}{n} \sum_{r=1}^{n} r a_r \\
&= \frac{n+1}{n} a_{nn}\lambda_n t_n + \sum_{v=1}^{n-1} \frac{v+1}{v} \Delta_v \left(\hat{a}_{nv}\right) \lambda_v t_v \\
&+ \sum_{v=1}^{n-1} \frac{v+1}{v} \hat{a}_{n,v+1} \Delta\lambda_v t_v + \sum_{v=1}^{n-1} \hat{a}_{n,v+1} \lambda_{v+1} \frac{t_v}{v} \\
&= I_{n,1} + I_{n,2} + I_{n,3} + I_{n,4}.
\end{aligned}$$

To complete the proof of Theorem 2, by Minkowski's inequality, it is enough to show that

$$\sum_{n=1}^{\infty} \varphi_n^{k-1} \mid I_{n,r} \mid^k < \infty, \quad for \quad r = 1, 2, 3, 4.$$

First, by using Abel's transformation, we have

$$\begin{aligned}
\sum_{n=1}^{m} \varphi_n^{k-1} \mid I_{n,1} \mid^k &= O(1) \sum_{n=1}^{m} \varphi_n^{k-1} a_{nn}^k |\lambda_n|^k |t_n|^k \\
&= O(1) \sum_{n=1}^{m} \varphi_n^{k-1} \left(\frac{p_n}{P_n}\right)^k |\lambda_n|^{k-1} |\lambda_n| |t_n|^k \\
&= O(1) \sum_{n=1}^{m} \varphi_n^{k-1} \left(\frac{p_n}{P_n}\right)^k |\lambda_n| |t_n|^k \\
&= O(1) \sum_{n=1}^{m-1} \Delta|\lambda_n| \sum_{v=1}^{n} \varphi_v^{k-1} \left(\frac{p_v}{P_v}\right)^k |t_v|^k \\
&+ O(1) |\lambda_m| \sum_{n=1}^{m} \varphi_n^{k-1} \left(\frac{p_n}{P_n}\right)^k |t_n|^k
\end{aligned}$$

$$
\begin{aligned}
&= O(1)\sum_{n=1}^{m-1}|\Delta\lambda_n|X_n + O(1)|\lambda_m|X_m \\
&= O(1)\sum_{n=1}^{m-1}\beta_n X_n + O(1)|\lambda_m|X_m \\
&= O(1) \quad as \quad m\to\infty,
\end{aligned}
$$

by virtue of the hypotheses of Theorem 2 and Lemma 1.

Now, applying Hölder's inequality with indices k and k', where $k>1$ and $\frac{1}{k}+\frac{1}{k'}=1$, we have

$$
\begin{aligned}
\sum_{n=2}^{m+1}\varphi_n^{k-1}\mid I_{n,2}\mid^k &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|\,|\lambda_v|\,|t_v|\right)^k \\
&= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|\,|\lambda_v|^k\,|t_v|^k\right)\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|\right)^{k-1}.
\end{aligned}
$$

Here, by (1) and (2), we have

$$\Delta_v(\hat{a}_{nv}) = \hat{a}_{nv}-\hat{a}_{n,v+1} = \bar{a}_{nv}-\bar{a}_{n-1,v}-\bar{a}_{n,v+1}+\bar{a}_{n-1,v+1} = a_{nv}-a_{n-1,v}.$$

Then, by using (1), (12) and (13), we get

$$\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})| = \sum_{v=1}^{n-1}(a_{n-1,v}-a_{nv}) \le a_{nn}.$$

Thus, we have

$$
\begin{aligned}
\sum_{n=2}^{m+1}\varphi_n^{k-1}\mid I_{n,2}\mid^k &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}a_{nn}^{k-1}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|\,|\lambda_v|^k\,|t_v|^k\right) \\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|\,|\lambda_v|^k\,|t_v|^k\right) \\
&= O(1)\sum_{v=1}^{m}|\lambda_v|^k|t_v|^k\sum_{n=v+1}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}|\Delta_v(\hat{a}_{nv})| \\
&= O(1)\sum_{v=1}^{m}\left(\frac{\varphi_v p_v}{P_v}\right)^{k-1}|\lambda_v|^k|t_v|^k\sum_{n=v+1}^{m+1}|\Delta_v(\hat{a}_{nv})|
\end{aligned}
$$

$$
\begin{aligned}
&= O(1)\sum_{v=1}^{m}\left(\frac{\varphi_v p_v}{P_v}\right)^{k-1}|\lambda_v|^{k-1}|\lambda_v||t_v|^k a_{vv}\\
&= O(1)\sum_{v=1}^{m}\varphi_v^{k-1}\left(\frac{p_v}{P_v}\right)^{k}|\lambda_v|\,|t_v|^k\\
&= O(1) \quad as \quad m\to\infty,
\end{aligned}
$$

as in $I_{n,1}$.

Now, again using Hölder's inequality, we have

$$
\begin{aligned}
\sum_{n=2}^{m+1}\varphi_n^{k-1}\mid I_{n,3}\mid^k &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\Delta\lambda_v||t_v|\right)^k\\
&= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|\beta_v|t_v|^k\right)\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|\beta_v\right)^{k-1}\\
&= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}a_{nn}^{k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|\beta_v|t_v|^k\right)\\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|\beta_v|t_v|^k\right)\\
&= O(1)\sum_{v=1}^{m}\beta_v|t_v|^k\sum_{n=v+1}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}|\hat{a}_{n,v+1}|\\
&= O(1)\sum_{v=1}^{m}\left(\frac{\varphi_v p_v}{P_v}\right)^{k-1}\beta_v|t_v|^k\sum_{n=v+1}^{m+1}|\hat{a}_{n,v+1}|.
\end{aligned}
$$

Here, by (1), (2), (12) and (13), we have

$$
\sum_{n=v+1}^{m+1}|\hat{a}_{n,v+1}| = \sum_{n=v+1}^{m+1}\sum_{i=0}^{v}(a_{n-1,i}-a_{ni}) \le 1.
$$

Thus, we get

$$
\begin{aligned}
\sum_{n=2}^{m+1}\varphi_n^{k-1}\mid I_{n,3}\mid^k &= O(1)\sum_{v=1}^{m}\varphi_v^{k-1}\left(\frac{p_v}{P_v}\right)^{k-1}v\beta_v\frac{1}{v}|t_v|^k\\
&= O(1)\sum_{v=1}^{m-1}\Delta(v\beta_v)\sum_{r=1}^{v}\varphi_r^{k-1}\left(\frac{p_r}{P_r}\right)^{k-1}\frac{1}{r}|t_r|^k
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad O(1)m\beta_m \sum_{v=1}^{m} \varphi_v^{k-1} \left(\frac{p_v}{P_v}\right)^{k-1} \frac{1}{v}|t_v|^k \\
&= \quad O(1) \sum_{v=1}^{m-1} v|\Delta\beta_v|X_v + O(1) \sum_{v=1}^{m-1} \beta_v X_v + O(1)m\beta_m X_m \\
&= \quad O(1) \quad as \quad m \to \infty,
\end{aligned}
$$

by virtue of the hypotheses of Theorem 2 and Lemma 1.

Again, using Hölder's inequality, we have

$$
\begin{aligned}
\sum_{n=2}^{m+1} \varphi_n^{k-1}|I_{n,4}|^k &\leq \sum_{n=2}^{m+1} \varphi_n^{k-1} \left(\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|}{v}\right)^k \\
&\leq \sum_{n=2}^{m+1} \varphi_n^{k-1} \left(\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|^k}{v}\right) \left(\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}|\frac{|\lambda_{v+1}|}{v}\right)^{k-1} \\
&\leq \sum_{n=2}^{m+1} \varphi_n^{k-1} a_{nn}^{k-1} \left(\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|^k}{v}\right) \left(\sum_{v=1}^{n-1} \frac{|\lambda_{v+1}|}{v}\right)^{k-1} \\
&= O(1) \sum_{n=2}^{m+1} \varphi_n^{k-1} \left(\frac{p_n}{P_n}\right)^{k-1} \left(\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|^k}{v}\right) \\
&= O(1) \sum_{n=2}^{m+1} \left(\frac{\varphi_n p_n}{P_n}\right)^{k-1} \left(\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|^k}{v}\right) \\
&= O(1) \sum_{v=1}^{m} |\lambda_{v+1}|\frac{|t_v|}{v}^k \sum_{n=v+1}^{m+1} \left(\frac{\varphi_n p_n}{P_n}\right)^{k-1} |\hat{a}_{n,v+1}| \\
&= O(1) \sum_{v=1}^{m} \left(\frac{\varphi_v p_v}{P_v}\right)^{k-1} |\lambda_{v+1}|\frac{|t_v|}{v}^k \sum_{n=v+1}^{m+1} |\hat{a}_{n,v+1}| \\
&= O(1) \sum_{v=1}^{m} \varphi_v^{k-1} \left(\frac{p_v}{P_v}\right)^{k-1} |\lambda_{v+1}|\frac{|t_v|}{v}^k \\
&= O(1) \sum_{v=1}^{m-1} |\Delta\lambda_{v+1}| \sum_{r=1}^{v} \varphi_r^{k-1} \left(\frac{p_r}{P_r}\right)^{k-1} \frac{1}{r}|t_r|^k \\
&+ O(1)|\lambda_{m+1}| \sum_{v=1}^{m} \varphi_v^{k-1} \left(\frac{p_v}{P_v}\right)^{k-1} \frac{1}{v}|t_v|^k
\end{aligned}
$$

$$
\begin{aligned}
&= O(1)\sum_{v=1}^{m-1}\beta_{v+1}X_{v+1} + O(1)|\lambda_{m+1}|X_{m+1}\\
&= O(1) \quad as \quad m\to\infty,
\end{aligned}
$$

by virtue of hypotheses of Theorem 2 and Lemma 1.

This completes the proof of Theorem 2.

5. Corollaries

Corollary 1. *If we take $\varphi_n = \frac{P_n}{p_n}$ in Theorem 2, then we get a known theorem dealing with $|A, p_n|_k$ summability of the series $\sum a_n\lambda_n$ (see [15]).*

Corollary 2. *If we take $\varphi_n = n$, $a_{nv} = \frac{p_v}{P_n}$ and $p_n = 1$ for all values of n in Theorem 2, then we get a result for $|C, 1|_k$ summability of the series $\sum a_n\lambda_n$.*

6. Conclusion

This study has a number of direct applications in rectification of signals in FIR filter (Finite impulse response filter) and IIR filter (Infinite impulse response filter). So, the absolute summability methods have potential in dealing with the problems based on infinite series.

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