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Investigation of Goursat-Darboux System with Integral Boundary Conditions

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Abstract. In this paper, we study a boundary value problem for hyperbolic nonlinear differential equations with integral boundary conditions. We first construct the Green function for reducing this boundary value problem to the corresponding integral equation. Furthermore, using the Banach principle of contraction mappings, we prove that the solution of the obtained integral boundary value problem exists and is unique.

Key Words and Phrases: nonlocal problem, integral boundary condition, Goursat-Darboux system.

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1. Introduction

In last decades, nonlocal problems for partial equations have been actively studied by many mathematicians. (see, e.i, [1, 2, 3]). The study of nonlocal problems is caused by both theoretical interest and practical necessity. This is due to the fact that mathematical models of various physical, chemical, biological and ecological processes often represent the problems in which, instead of classical boundary conditions, some relationship is given between the values of the desired function at the boundary of the domain and inside it. Such problems arise in the study of phenomena related to plasma physics, heat propagation [1], demography, mathematical biology [2, 3], etc.

It should be noted that nonlocal problems with integral boundary conditions for hyperbolic equations and also the existence and uniqueness of their solutions have been studied in [4–21]. Conditions of classical and general consistency of problems with integral conditions for second order hyperbolic equations have been obtained as a result of these works. Furthermore, Goursat-Darboux nonlocal

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problem for one-dimensional hyperbolic equation with a mixed derivative has been considered in [6, 7, 8, 9, 15, 16], where sufficient conditions for the existence of the classical solution of the problem under consideration have been obtained in terms of the coefficients and the kernel in the integral condition.

Usually, in practical problems it is assumed that the coefficients of the equation are continuous. There arise nonlocal problems with integral conditions for a system of hyperbolic equations in the process of studying boundary value problems with integral conditions for higher order partial equations [2, 7, 9]. A nonlocal problem with an integral condition in one of the variables for a system of hyperbolic equations with a mixed derivative has been studied in [9, 13]. Necessary and sufficient conditions of well-defined consistency of the given problem have been established in terms of initial data and the solution algorithms have been suggested.

2. Problem statement

In this paper, we consider a nonlocal problem with integral boundary conditions for the Goursat-Darboux system in the domain $Q = [0,T] \times [0,l]$:

$$y_{tx} = f(t,x,y), \quad (t,x) \in Q, \tag{1}$$

$$\int_0^T y(t,x)\,dt = \varphi(x), \quad x \in [0,\ell], \tag{2}$$

$$\int_0^\ell y(t,x)\,dx = \psi(t), \quad t \in [0,T], \tag{3}$$

where $y(t,x) = col\,(y_1(t,x), y_2(t,x), \ldots y_n(t,x))$ is an unknown n-dimensional vector-function, $f(t,x,y)$ is continuous on $Q \times R^n$, n-dimensional vector-functions $\varphi(x)$ and $\psi(t)$ are continuously-differentiable on $[0,l], [0,T\ ]$, respectively.

It is assumed that the functions $\varphi(x)$ and $\psi(t)$ satisfy the agreement condition

$$\int_0^T \psi(t)\,dt = \int_0^\ell \varphi(x)\,dx.$$

Note that problems with integral conditions for hyperbolic type equations have been studied in [4,5,8,9,12-21], where the conditions of classical, general consistency of problems with integral conditions have been established for second order hyperbolic equations.

Let $C(Q,R^n)$ be a space of vector-functions continuous on Q with the norm $\|y\| = \max_{(t,x)\in Q} |y(t,x)|$, $|y(t,x)| = \sqrt{y_1^2(t,x) + \ldots y_n^2(t,x)}$.

The function $y(t,x) \in C(Q,R^n)$ with partial derivatives $y_t(t,x) \in C(Q,R^n)$, $y_x(t,x) \in C(Q,R^n)$, $y_{tz}(t,x) \in C(Q,R^n)$ is said to be a classical solution of problem (1)-(3) if it satisfies the system of equations (1) and integral conditions (2)-(3).

There are chemico-physical and ecological processes in the nature, main characteristics of which cannot be measured directly. Goursat-Darboux problems with nonlocal conditions arise during mathematical modeling of such processes and it represents information on mean values or multipoint conditions. Such problems can arise in the study of phenomena related to plasma physics, heat propagation, demography, mathematical biology, etc.

3. Main results

In this paper, the Green function is constructed for the first time for the problem (1)-(3), which is reduced to an equivalent integral equation. Further, using the Banach contraction mapping principle, sufficient conditions of classical consistency of the given problem are established.

Theorem 1. *Problem* (1)-(3) *is equivalent to the following integral equation:*

$$y(t,x) = \frac{1}{\ell}\psi(t) + \frac{1}{T}\varphi(x) - \frac{1}{\ell T}A + \int_0^T \int_0^\ell G(t,x,\tau,s)\, f(\tau,s,y)\, dtds,$$

where

$$G(t,x,\tau,s) = E \times \begin{cases} \frac{1}{\ell T} s\tau, & 0 \le \tau \le t, \quad 0 \le s \le x, \\ -\frac{1}{\ell T} s(T-\tau), & 0 \le s \le x, \quad t < \tau \le T, \\ -\frac{1}{T\ell}\tau(\ell - s), & 0 \le \tau \le t, \ x < s \le \ell, \\ \frac{1}{T\ell}(T-\tau)(\ell - s), & x < s \le \ell, \ t < \tau \le T. \end{cases}$$

$A = \int_0^T \psi(t)\,dt = \int_0^\ell \varphi(x)\,dx = const$; *$E$ is an identity matrix of dimension $n \times n$.*

Proof. We will look for any solution of equation (1) in the form

$$y(t,x) = a(t) + b(x) + \int_0^t \int_0^x f(t,s,y(t,s))dtds, \tag{4}$$

where $a(t)$ and $b(x)$ are unknown continuous functions defined on the segments $[0,\ T]$, $[0,\ \ell]$, respectively. Let the function determined by equality (4) satisfy conditions (2) and (3). Then

$$\int_0^T a(t)\,dt + Tb(x) + \int_0^T \left(\int_0^t \int_0^x f(t,s,y(t,s))\,dtds \right) dt = \varphi(x), \ x \in [0,\ell], \tag{5}$$

$$\ell a\left(t\right)+\int_0^\ell b\left(x\right)dx+\int_0^\ell \left(\int_0^t\int_0^x f\left(t,s,y\left(t,s\right)\right)dtds\right)dx=\psi\left(t\right),t\in\left[0,T\right]. \quad (6)$$

Without loss of generality, we assume that the relation

$$\int_0^T a\left(t\right)dt=0$$

is valid.

From equality (5) we obtain the following relation:

$$b\left(x\right)=\frac{1}{T}\varphi\left(x\right)-\frac{1}{T}\int_0^T\left(\int_0^t\int_0^x f\left(\tau,s,y\left(\tau,s\right)\right)d\tau ds\right)dt,\ x\in\left[0,\ell\right]\ . \quad (7)$$

Taking into account (7) in (6), we get

$$a\left(t\right)=\frac{1}{\ell}\psi\left(t\right)+\frac{1}{T\ell}\int_0^T\int_0^\ell\left(\int_0^t\int_0^x f\left(\tau,s,y\left(\tau,s\right)\right)d\tau ds\right)dtdx-$$
$$-\frac{1}{\ell}\int_0^\ell\left(\int_0^t\int_0^x f\left(\tau,s,y\left(\tau,s\right)\right)d\tau ds\right)dx-A,\ t\in\left[0,T\right]. \quad (8)$$

Taking into account (7) and (8) in (4), we get

$$y\left(t,x\right)=\frac{1}{\ell}\psi\left(t\right)+\frac{1}{T}\varphi\left(x\right)-\frac{1}{T\ell}A-\frac{1}{\ell}\int_0^\ell\left(\int_0^t\int_0^x f\left(\tau,s,y\left(\tau,s\right)\right)d\tau ds\right)dx-$$
$$-\frac{1}{T}\int_0^T\left(\int_0^t\int_0^x f\left(\tau,s,y\left(\tau,s\right)\right)d\tau ds\right)dt+$$
$$+\frac{1}{T\ell}\int_0^T\int_0^\ell\left(\int_0^t\int_0^x f\left(\tau,s,y\left(\tau,s\right)\right)d\tau ds\right)dtdx+$$
$$+\int_0^t\int_0^x f\left(\tau,s,y\left(\tau,s\right)\right)d\tau ds,\left(t,x\right)\in Q. \quad (9)$$

Make some transformations in equality (9):

$$\int_0^\ell\left(\int_0^t\int_0^x f\left(\tau,s,y\left(\tau,s\right)\right)d\tau ds\right)dx=\int_0^\ell\int_0^t\left(\ell-x\right)f\left(\tau,s,y\left(\tau,s\right)\right)d\tau dx.$$

$$\int_0^T\left(\int_0^t\int_0^x f\left(\tau,s,y\left(\tau,s\right)\right)d\tau ds\right)dt=\int_0^T\int_0^x\left(T-t\right)f\left(\tau,s,y\left(\tau,s\right)\right)dtds$$

$$\int_0^T\int_0^\ell\left(\int_0^t\int_0^x f\left(\tau,s,y\left(\tau,s\right)\right)d\tau ds\right)dtdx=$$

$$= \int_0^T \int_0^\ell (T-t)(\ell-x) f(\tau,s,y(\tau,s))\, d\tau ds.$$

Taking into account these equalities in (9), we obtain:

$$y(t,x) = \frac{1}{\ell}\psi(t) + \frac{1}{T}\varphi(x) - \frac{1}{T\ell}A+$$

$$+\int_0^t \int_0^x \left[\frac{1}{\ell T}(\ell-s)(T-\tau) + E - \frac{1}{\ell}(\ell-s) - \frac{1}{T}(T-\tau)\right] f(\tau,s,y(\tau,s))\, d\tau ds+$$

$$+\int_x^\ell \int_0^t \left[\frac{1}{\ell T}(s-\ell)(T-\tau) - \frac{1}{\ell}(\ell-s)\right] f(\tau,s,y(\tau,s))\, d\tau ds$$

$$+\int_t^T \int_0^x \left[\frac{1}{T\ell}(\ell-s)(T-t) - \frac{1}{T}(T-\tau)\right] f(\tau,s,y(\tau,s))\, d\tau ds$$

$$+\int_t^T \int_x^\ell \frac{1}{T\ell}(\ell-s)(T-\tau) f(\tau,s,y(\tau,s))\, d\tau ds, (t,x)\in Q. \qquad (10)$$

Taking into consideration the relations

$$E\left[1 + \frac{1}{T\ell}(\ell-s)(T-t) - \frac{1}{\ell}(\ell-s) - \frac{1}{T}(T-\tau)\right] = E\frac{1}{\ell T}st,$$

$$E\left[\frac{1}{\ell T}(\ell-s)(T-\tau) - \frac{1}{T}(T-\tau)\right] = -E\frac{1}{T\ell}s(T-\tau),$$

$$E\left[\frac{1}{\ell T}(\ell-s)(T-\tau) - \frac{1}{\ell}(\ell-s)\right] = -E\frac{1}{T\ell}\tau(\ell-s),$$

we can rewrite equality (4) in the following form:

$$y(t,x) = \frac{1}{\ell}\psi(t) + \frac{1}{T}\varphi(x) - \frac{1}{T\ell}A + \int_0^t \int_0^x \frac{1}{\ell T}\tau s f(\tau,s,y(\tau,s))\, d\tau ds+$$

$$+\int_x^\ell \int_0^t \left[-\frac{1}{T\ell}\tau(\ell-s)\right] f(\tau,s,y(\tau,s))\, d\tau ds+$$

$$+\int_t^T \int_0^x \left[-\frac{1}{T\ell}s(T-\tau)\right] f(\tau,s,y(\tau,s))\, d\tau ds+$$

$$+\int_t^T \int_x^\ell \frac{1}{T\ell}(\ell-s)(T-\tau) f(\tau,s,y(\tau,s))\, d\tau ds. \qquad (11)$$

We'll consider the matrix function $G(t,\ x,\ t,\ s)$ in this equation in order to prove the first part of the theorem. At first we'll show that the function defined by equality (11) is the solution of boundary value problem (1)-(3). Let's calculate its derivative with respect to t and x:

$$y_{tx}(t,x) = \frac{\partial^2}{\partial t \partial x}\left[\frac{1}{\ell}\psi(t) + \frac{1}{T}f(x) - \frac{1}{T\ell}A\right] +$$

$$+\frac{\partial^2}{\partial t \partial x}\left[\int_0^t \int_0^x \frac{1}{\ell T}\tau s f(\tau, s, y(\tau, s))\, d\tau ds\right] +$$

$$+\frac{\partial^2}{\partial t \partial x}\left[\int_x^\ell \int_0^t \left[-\frac{1}{T\ell}\tau(\ell - s)\right] f(\tau, s, y(\tau, s))\, d\tau ds\right] +$$

$$+\frac{\partial^2}{\partial t \partial x}\left[\int_t^T \int_0^x \left[-\frac{1}{T\ell}s(T-\tau)\right] f(\tau, s, y(\tau, s))\, d\tau ds\right] +$$

$$+\frac{\partial^2}{\partial t \partial x}\left[\int_t^T \int_x^\ell \frac{1}{T\ell}(\ell - s)(T-\tau) f(\tau, s, y(\tau, s))\, d\tau ds\right] =$$

$$= \frac{1}{\ell T}(tx + t\ell - tx + xT - xt + \ell T - t\ell - xT + tx) \times$$

$$\times f(t, x, y(t,x))\, d\tau ds = f(t, x, y(t,x)).$$

Now we'll prove the second part of the theorem. Let's show that the function defined by equality (11) satisfies conditions (2) and (3).

$$\int_0^T y(t,x)\, dt = \int_0^T \left[\frac{1}{\ell}\psi(t) + \frac{1}{T}\varphi(x) - \frac{1}{\ell T}A -\right.$$

$$-\frac{1}{\ell}\int_0^\ell \left(\int_0^t \int_0^x f(\tau, s, y(\tau, s))\, d\tau ds\right) dx -$$

$$-\frac{1}{T}\int_0^T \left(\int_0^t \int_0^x f(\tau, s, y(\tau, s))\, d\tau ds\right) dt +$$

$$+\frac{1}{\ell T}\int_0^T \left(\int_0^\ell \int_0^t f(\tau, s, y(\tau, s))\, d\tau ds\right) dt dx +$$

$$\left. + \int_0^t \int_0^x f(\tau, s, y(\tau, s))\, d\tau ds\right] dt =$$

$$= \frac{1}{\ell}\int_0^T \psi(t)\, dt + \varphi(x) - \frac{1}{\ell}\int_0^T \psi(t)\, dt -$$

$$-\frac{1}{\ell}\int_0^T \int_0^\ell \left(\int_0^t \int_0^x f(\tau, s, y(\tau, s))\, d\tau ds\right) dt dx -$$

$$-\int_0^T \left(\int_0^t \int_0^x f\left(\tau, s, y\left(\tau, s\right)\right) d\tau ds \right) dt +$$

$$+\frac{1}{\ell} \int_0^T \int_0^\ell \left(\int_0^t \int_0^x f\left(\tau, s, y\left(\tau, s\right)\right) d\tau ds \right) dt dx +$$

$$+\int_0^T \left(\int_0^t \int_0^x f\left(\tau, s, y\left(\tau, s\right)\right) d\tau ds \right) dt = \varphi\left(x\right).$$

We can similarly show that the condition

$$\int_0^\ell y\left(t, x\right) dx = \psi\left(t\right), \ t \in [0, T],$$

is also satisfied.

Thus, Theorem 1 is completely proved. ◀

To prove the uniqueness of the solution of the stated problem, we define the operator $P : C\left(Q; R^n\right) \to C\left(Q; R^n\right)$ as

$$P\left(z\right) = \frac{1}{\ell}\psi\left(t\right) + \frac{1}{T}\varphi\left(x\right) - \frac{1}{\ell T}A + \int_0^T \int_0^\ell G\left(t, x, \tau, s\right) f\left(\tau, s, z\right) dt ds.$$

It is known that the problem (1)-(3) is equivalent to the fixed point problem $z = Pz$. So, problem (1)-(3) has a solution if and only if the operator P has a fixed point.

Theorem 2. *Assume that the condition*

$$\left|f\left(t, x, z_2\right) - f(t, x, z_1)\right| \le M\left|z_2 - z_1\right| \tag{12}$$

is satisfied for each $(t, x) \in Q$ *and for all* $z_1, \ z_2 \in R^n$, *the constant* $M \ge 0$, *and*

$$L = \ell TSM < 1\ , \tag{13}$$

where

$$S = \max_{Q \times Q} \left\|G\left(t, x, \tau, s\right)\right\|\ .$$

Then boundary value problem (1)-(3) *has a unique solution on* Q.

Proof. Denote

$$N = \max_{Q} \left| \frac{1}{\ell}\psi\left(t\right) + \frac{1}{T}\varphi\left(x\right) - \frac{1}{\ell T}A \right|\ ,$$

$$\max_{(t,x) \in Q} |f(t, x, 0| = M_f\ ,$$

and choose $r \geq \frac{\|N\|+M_f TS}{1-L}$. We'll prove that $PB_r \subset B_r$, where

$$B_r = \{x \in C(Q, R^n) : \|z\| \leq r\}.$$

For $z \in B_r$ we have

$$\|Pz(t,x)\| \leq \|N\| + \int_0^T \int_0^\ell |G(t,z,\tau,s)| \left(|f(\tau,s,z(t,s)) - f(\tau,s,0)| + \right.$$

$$\left. + |f(t,s,0)|\right) d\tau ds \leq \|N\| + S\int_0^T \int_0^\ell (M|z| + M_f)\, dtdx \leq$$

$$\leq \|N\| + SMrT\ell + M_f T\ell S \leq \frac{\|N\| + M_f TS}{1-L} \leq r.$$

Further, by (12), for any $z_1, z_2 \in B_r$

$$|Pz_2 - Pz_1| \leq \int_0^T \int_0^\ell |G(t,x,\tau,s)(f(\tau,x,z_2(\tau,s)) - f(t,x,z_1(\tau,s)|\, d\tau ds \leq$$

$$\leq S\int_0^T \int_0^\ell M|z_2(t,x) - z_1(t,x)|\, dtdx \leq MST\ell \max_{Q\times Q} |z_2(t,x) - z_1(t,x)|$$

$$\leq MST\ell \|z_2 - z_1\|$$

is valid, or

$$\|Pz_2 - Pz_1\| \leq L\|z_2 - z_1\|.$$

It is clear that by condition (13) P is a contraction operator. Thus, boundary value problem (1)-(3) has a unique solution. ◀

4. Example

Let's give an example illustrating the main results obtained in this paper. Consider the following system of differential equations with an integral boundary conditions:

$$\left\{ \begin{array}{l} y_{1tx} = 0,1 cos y_2, \\ y_{2tx} = \frac{|y_1|}{(9+e^{tx})(1+|y_1|)} \end{array} \right., \qquad (t,x) \in [0,1] \times [0,1], \qquad (14)$$

$$\left\{ \begin{array}{l} \int_0^1 y_1(t,x)\, dt = x, \int_0^1 y_2(t,x)\, dt = x^2, \\ \int_0^1 y_1(t,x)\, dx = t, \int_0^1 y_2(t,x)\, dx = t^2. \end{array} \right. \qquad (15)$$

Obviously, the agreement condition is satisfied. Condition (13) is satisfied due to (12) and $G_{max} \leq 1, \;\; M = 0,1$. Consequently,

$$L = G_{max} M T \ell = 1 \cdot 0,1 = 0,1 < 1.$$

So, by Theorem 2, boundary value problem (14)-(15) has a unique solution on $[0,1] \times [0,1]$.

5. Conclusion

In this paper, the existence and uniqueness of solutions for nonlinear hyperbolic differential equations with integral boundary conditions are established. Note that the method introduced here can be successfully used in more complicated problems for hyperbolic differential equation. For example, we can consider the following problem:

$$y_{tx} = f\left(t, x, y\right), \quad (t,x) \in Q \ ,$$

with integral boundary conditions

$$\int_0^T n(t) y\left(t, x\right) dt = \varphi\left(x\right), \quad x \in [0, \ell]\,,$$

$$\int_0^\ell m\left(x\right) y\left(t, x\right) dx = \psi\left(t\right), \quad t \in [0, T]\,.$$

Here $n\left(t\right), m\left(x\right) \in R^{n \times n}$ are the given matrices; $\varphi\left(x\right), x \in [0, \ell]\,, \psi\left(t\right), \;\; t \in [0, T]$ are the given functions, and $det \int\limits_0^T n(t) dt \neq 0, \;\; det \int\limits_0^\ell m\left(x\right) dx \neq 0$.

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